The Pomeron: past, present and future

Raju Venugopalan

Stony Brook University
&
Brookhaven National Laboratory

Simons Confinement Collaboration Inaugural Workshop, September 8-10, 2022
Effective field theories of QCD to understand data from collider experiments: RHIC, LHC, EIC,...
Preliminary remarks

Vast pre- and post- QCD literature on the Pomeron
– impossible to survey in this talk

The Pomeron is “God Particle” in a certain sense:
- No one really knows what it is people have very differing ideas on what it is
- Its not clear it is a robust concept in QCD
- It is endlessly fascinating

I will also not discuss the interesting literature (BPST Pomeron and descendants)
on the Pomeron in holography

My perspective is a narrow one but hopefully of interest in particular with regard to matching non-perturbative approaches to the UV
Landscape of the strong interaction

Aschenauer et al., arXiv:1708.01527
High energy cross-sections: the Pomeron

Total cross-sections across three orders of magnitude in energy (SPS -> LHC) simply described in terms of Pomeron and Reggeon trajectories:

Scattering amplitude $A(s, t) = s^{\alpha(t)}$ with $\alpha(t) = 1 + \varepsilon + \alpha' t$

Pomeron: t-channel exchange (corresponding to a pole in the t-j plane) with vacuum quantum numbers dominates total hadron-hadron cross-sections

Intercept $\varepsilon_P = 0.11$ String tension $\alpha' = 0.165$ GeV$^{-2}$
High energy cross-sections: Pomerons+Reggeons

Chew-Frautschi plot of Reggeon trajectories for +ve t

\[ J(M^2) = \alpha(0) + \alpha' M^2 \]

\[ \alpha' = \frac{1}{2\pi\sigma} \]

<table>
<thead>
<tr>
<th>trajectory</th>
<th>(\sqrt{\sigma}/\text{MeV} )</th>
<th>(\Delta J )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi, \rho_1, \ldots)</td>
<td>469(6)</td>
<td>0.06</td>
</tr>
<tr>
<td>(\rho, \omega_1, \ldots)</td>
<td>429(2)</td>
<td>0.03</td>
</tr>
<tr>
<td>(\omega, f_2, \ldots)</td>
<td>436(8)</td>
<td>0.12</td>
</tr>
<tr>
<td>(\phi, f_2', \ldots)</td>
<td>437(5)</td>
<td>0.06</td>
</tr>
<tr>
<td>(K, K_1, \ldots)</td>
<td>480(4)</td>
<td>0.04</td>
</tr>
<tr>
<td>(K^<em>, K^</em>_2, \ldots)</td>
<td>424(5)</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Relativistic quark model

Reggeon field theory:
Veneziano dual resonance model relating s and t channel S-matrix unitarity provided much of the powerful initial (pre+post-QCD) motivation for string theory and incipient relativistic quark model for confinement

Veneziano, Nuovo Cim. 57, 190 (1968)
Unsolved problem: What is the Pomeron in QCD. Is it a robust object?

If so, unitarity (Froissart bound) + (more mundane) diffractive dissociation demand multi-Pomeron dynamics/exchanges

All evidence points to Pomeron dynamics being dominated by glue

Simplest picture of the Pomeron – color singlet 2 gluon exchange

It’s “Odderon” (C=-1 partner, responsible for the difference in p+p and p+p̅ cross-sections)

has quantum numbers #’s of 3-gluon exchange

Low, Nussinov (1975)
Lukaszuk, Nicolescu (1973)
Pomeron in perturbative QCD: BFKL Pomeron

Computing $2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation

\[ S = (P_A + P_B)^2 = 2P_A \cdot P_B \]

\[ S_i = (k_i + k_{i+1})^2 = 2k_i \cdot k_{i+1} \]

\[ k_i = (q_i - q_{i+1}) \quad k_0 = P'_A, k_{n+1} = P'_B \]

**Multi-Regge limit**: (MRK)

$S \gg s_i \sim s_2 \sim \ldots \sim s_{n+1} \gg q_i^2 \sim q_2^2 \sim \ldots \sim q_{n+1}^2$

\[ q_i^2 = -k_i^2 \quad a_i = q_i P_B + P_i P_A + k_i \]

\[ S_i \sum_{i=1}^{n+1} (k_i^2) = S_1 + S_2 + \ldots + S_{n+1} \quad k_i \cdot P_A = k_i \cdot P_B = 0 \]

Alternatively express in terms of rapidities

\[ q_i \sim (k_{i+1} \cdot y_i - k_i \cdot x_i, -k_i) \]

\[ q_i^2 \sim -k_i^2 = \xi_i \]

\[ y_0 \gg y_1 \gg y_2 \ldots \gg y_{n+1} \quad y_i \sim \ln \left( \frac{x_i}{\xi_i} \right) \]

BFKL Hamiltonian:

Remarkable properties: holomorphic separability;

generalization to an integrable model;

Large body of beautiful work in N=4 SUSY

BFKL Pomeron

BFKL: Balitsky,Fadin,Kuraev,Lipatov (1976-1978)
2 → N + 2 amplitude in the Regge limit: the BFKL equation

To build in real and virtual corrections to all orders in $\alpha_s$, first focus on one rung of 2 → N+2 ladder

**Building blocks: Real Emission**

**Lipatov vertex**

$$C(q_{i+1}, q_i) = -q_{i+1} - q_i$$

$$+ P_A \left( \frac{2 \cdot \xi}{z_i} + B_i \right) - P_B \left( \frac{2 \cdot \xi}{z_i} + d_i \right)$$

**Non-local gauge invariant Ward identity**:

$$k_i \cdot C = 0$$

**BFKL pedagogical review:**

DelDuca, hep-ph/9503226
2 → N + 2 amplitude in the Regge limit of QCD: the BFKL equation

Building Blocks: virtual corrections

\[ \frac{1}{t} \rightarrow \frac{1}{t} \frac{\ln s}{-t} \]

with \( \alpha(t) \propto \ln \frac{-t}{m^2} \)

Reggeization ansatz:

\[ \alpha^{(2)}(t) = \kappa^2 T \left( \frac{\mu^2}{-t} \right)^{2\epsilon} \left( \frac{\beta_0}{\epsilon^2} + \frac{\gamma_K^{(2)}}{8\epsilon} + \frac{\gamma_\Lambda^{(2)}}{2} + \zeta_2 \beta_0 \right) + O(\epsilon) \]

\( \gamma_K^{(2)} \): Two loop cusp anomalous dimension

\( \gamma_\Lambda^{(2)} \): Two loop wedge anomalous dimension

\( \frac{1}{t} \rightarrow \frac{1}{t^\epsilon} e^{\alpha(t) (y_i - y_j)} \)

Double log structure:

Sudakov form factor

→ infrared sensitive

\( \gamma_0 \): 2 → 2 amp. vanishes for \( \mu \rightarrow 0 \)

Fadin, hep-ph/9807528
2 → N + 2 amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons

\[ \text{BFKL in Mellin space} \]

\[ \omega_f(q_{1\perp}, q_{2\perp}) = \frac{1}{2} \delta^{(2)}(q_{1\perp} - q_{2\perp}) + (\mathcal{K} \ast f_\omega)(q_{1\perp}, q_{2\perp}) \]

\[ (\mathcal{K} \ast \Phi_{\nu n})(q_{\perp}) = \omega_{\nu n} \Phi_{\nu n}(q_{\perp}) \]

\[ \text{Im} A(\delta, t) \propto \sum_{m=0}^{\infty} (\alpha_s C_T)^{m+2} \]

\[ \times \int \prod_{l=1}^{n} \frac{d^2 y_i}{(2\pi)^2} \frac{n!}{\prod_{j=1}^{n+1} (2\pi)^2} \]

\[ \times 2i \delta \prod_{l=1}^{n+1} \frac{1}{(x_l - \alpha_l)(x_l + \alpha_l)} \]

\[ \times \prod_{m=1}^{n} (C_n C^m) [2m, 2m+1] \]

\[ \text{C}_T \text{is color factor} \]

\[ \text{Phase space factors} \]

\[ \text{Reggeized propagators on both sides of cut} \]

\[ \text{Product of Lipatov vertices} \]

\[ \text{Real and virtual corrections combine to cancel infrared divergence!} \]

\[ \text{Strongly violates Froissart bound} \]

\[ \text{Resummed NLO BFKL : } \lambda \approx 0.3 \]

\[ \mathcal{E}_{tot} = 2 \text{Im} \dot{A}(\delta, t = 0) \]

\[ = 8^\lambda \text{ with } \lambda = \frac{\alpha_s N_c \Lambda_{IR}^2}{\bar{\Lambda}} \]

\[ \approx 0.5 \text{ for } \alpha_s = 0.2 \]
2 → N + 2 amplitude in the Regge limit: the NLL BFKL equation

Regge factorization at NLL \([\alpha_S \ln(s/t)^n]\): Includes one loop corrections to the Lipatov vertex \(V^{(1)}\) and two loop corrections to the Regge trajectory \(\alpha^{(2)}\)

Beyond NLL:

Three reggeized gluon exchange corresponds to Regge cut in angular momentum plane – this can be computed

Multi-Regge limit of planar SYM \(\mathcal{N} = 4\):

At large t’Hooft coupling, AdS/CFT duality between amplitudes and minimal area surfaces with closed light-like polygon boundaries

Dual conformal tranformations → BDS ansatz;
rich mathematical structure of MHV amplitudes in MRK kinematics

Figures from excellent review of state-of-the art:
Del Duca, Dixon, arXiv:2203.13026

Fadin, Lipatov, hep-ph/9802290
Camici, Ciafaloni, hep-ph/8903389

BDS: Bern, Dixon, Smirnov
See for example, Dixon, Liu, Miczajka, arXiv:2110.11388
Breakdown of OPE: Multi-Pomeron and Reggeon exchange

Rapid BFKL growth leads to large phase-space occupancy $N$ at high energies → novel many-body gluodynamics

Partons recombine and screen – many-body “shadowing”

\[
N \rightarrow \frac{1}{\alpha_s} = \text{classicalization!}
\]

Gribov, Levin, Ryskin (1983)
Mueller, Qiu (1986)
Gluon saturation: classicalization and perturbative unitarization

s-channel "dipole" scattering picture
– more convenient for multi-pomeron interactions

\[ \sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[ 1 - \exp \left( -r_\perp^2 Q_s^2(x) \right) \right] \]

Emergent semi-hard scale \( Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda \)

\( r_\perp^2 Q_s^2 \ll 1 \) (\( \sigma \propto A \))

Color transparency

\( r_\perp^2 Q_s^2 \gg 1 \) (\( \sigma \propto A^{2/3} \))

Color opacity ("black disk")

QCD picture of observed "shadowing" at small x

Mueller, NPB415 (1994) 373
Mueller, Patel, hep-ph/9403256
Classicalization in the Regge limit: the Color Glass Condensate EFT

**Born-Oppenheimer separation** between fast and slow modes

**CGC**: Effective Field Theory of dynamical gluon fields coupled to strong color sources

- **Large x** \((P^+)\) modes: static, strong \((\sim 1/g)\) color sources \(\rho^a\)
- **Small x** \((k^+ \ll P^+)\) modes: fully dynamical gauge fields \(A_\mu^q\)

CGC review: Gelis,Iancu,Jalilian-Marian,RV:arXiv 1002.0333

\[
Z[j] = \int [d\rho] W_{A^+} [\rho] \left\{ \frac{\int_{\Lambda^+} [dA] \delta(A^+) e^{iS_{\Lambda^+} [A,\rho]} - \int j^i A_d}{\int_{\Lambda^+} [dA] \delta(A^+) e^{iS_{\Lambda^+} [A,\rho]}} \right\}
\]

\(W_{A^+}[\rho]\) : nonperturbative gauge invariant weight functional defined at initial \(x_0 = \Lambda^+/P^+\)

Explicit construction of effective action for large nuclei (large # of coherent color sources at small \(x\))

\(W_{A^+}[\rho] \rightarrow \int [d\rho] \exp \left( - \int d^2 x_{\perp} \left[ \frac{\rho^a\rho^a}{2\mu_A^2} - \frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa_A} \right] \right)\)

Classical saddle point is the non-Abelian Weizsacker-Williams field - demonstrates emergence of dimensionful \(Q^2_S\) scale \(\propto A^{1/3}\)

General all-order formalism: Cutkosky’s rules in strong fields

\[ 2 \text{Im} \sum_{\text{conn.}} V = \]

Propagators on Schwinger-Keldysh contour

Well-known example: Schwinger pair production in strong field QED

Simple understanding of “AGK cutting rules” of reggeon field theory as combinatorics of cut and uncut sub-graphs contributing to a given multiplicity

AGK: Abramovsky, Gribov, Kancheli

- Very general consequence of unitarity in strong fields
- Independent of language of Pomerons and Reggeons

RG hierarchy of many-body correlators in QCD

Multiple scattering or “shockwave” depending on gauge choice

\[ \frac{\partial}{\partial Y} \langle \mathcal{O}[\rho]\rangle_Y = \frac{1}{2} \int_{x,y} \frac{\partial}{\partial \rho^a(x)} \chi^{ab}_{x,y} \frac{\partial}{\partial \rho^b(y)} \mathcal{O}[\rho]\rangle_Y \]

“time”

“diffusion coefficient”: retarded Green function in strong background

Langevin diffusion of fuzz of “wee” partons in functional space of color fields

B-JIMWLK hierarchy for n-point Wilson line correlators:

corresponding JIMWLK Hamiltonian known to NLL accuracy

Balitsky, hep-ph/9509348
Iancu, Leonidov, McLerran, hep-ph/0011241
Inclusive DIS: dipole evolution in gluon shockwave background

B-JIMWLK RG eqn. for dipole correlator:

$$ \frac{\partial}{\partial Y} \langle \text{Tr}(V_x V_y^\dagger) \rangle_Y = -\frac{\alpha_S N_c}{2\pi^2} \int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2(z_\perp - y_\perp)^2} \langle \text{Tr}(V_x V_y^\dagger) - \frac{1}{N_c} \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y $$

Dipole factorization:

$$ \langle \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y \rightarrow \langle \text{Tr}(V_x V_z^\dagger) \rangle_Y \langle \text{Tr}(V_z V_y^\dagger) \rangle_Y \quad A >> 1, \ N_c \rightarrow \infty $$

Closed form expression is non-linear Balitsky-Kovchegov (BK) equation
- nearly equivalent to FKPP eqn. for pulled traveling wavefronts


Saturates cross-section for fixed impact parameter
The BFKL equation is the low density $V \approx 1 - ig\rho / \nabla T^2$ limit of the BK equation...

Y = Ln(1/x)
Implicit are a number of assumptions about confining dynamics: impact parameter dependence, factorization, fragmentation/hadronization
Colliding gluon shockwaves in a heavy-ion collision

Ab initio description

Thermalized soft bath of gluons at $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature: $T_i = \alpha_S^{2/5} Q_S$

$\tau_{\text{therm}} \propto \frac{(\log Q_S)^{2.5}}{Q_S} \rightarrow 0$ for $QS \rightarrow \infty$

Uncovering early-time dynamics of QGP important focus of RHIC & LHC heavy-ion experiments

Berges, Heller, Mazeliauskas, Venugopalan, Rev. Mod. Phys. 93, (2021) 035003

Colliding Color Glass Condensates

Quark-Gluon plasma
Back to the Pomeron...

Although the dipole cross-section saturates for fixed impact parameter at sufficiently high energies, the evolution kernel contains a Coulomb tail at large impact parameters – this violates the Froissart bound, which is unsurprising, since the latter requires the existence of a mass gap.

The outstanding question is how to map this BFKL/BK dynamics in the CGC EFT to strong coupling physics at large impact parameters? – is there a way (consistent with small x RG evolution of the UV) to incorporate confining dynamics.

A related issue is what sets the scale of the coefficient of the \( \ln^2 s \) term in the Froissart bound. Fits to data suggest that this scale is \( \sim 10 m_\pi \) consistent with lightest glueball mass/chiral scale \( 4\pi f_\pi \).
Landscape of the strong interaction

- Quarks and Gluons
- Q$^2$(GeV$^2$)
- Resolution
- Strongly Correlated Quark-Gluon Dynamics
- Linear evolution
- Non-linear evolution
- Perturbative
- Non-perturbative
- Confinement, Chiral Symmetry Breaking
- Lattice QCD
- High-Density Gluon Matter
- CGC EFT
- Pomerons
- Regge trajectories
- EFT of long strings?
EIC will have 1000 times the luminosity of HERA - clean, highly differential studies of pomeron dynamics
Also, first such studies in hard diffractive DIS off nuclei – is the pomeron flux universal?
Thanks for listening!
Effective Field Theory on Light Front

Explicit construction classical EFT in the Regge limit for large nuclei:

Gaussian stochastic distribution of k static color sources coherently coupled to gauge fields

\[ \mathcal{N} \int d m \, d n \, d m_{n} \, N_{m,n}^{(k)} : \int [d \rho] \exp \left( - \int d^{2} x_{\perp} \left[ \frac{\rho^{a} \rho^{a}}{2 \mu^{2}} - \frac{d_{abc} \rho^{a} \rho^{b} \rho^{c}}{\kappa_{A}} \right] \right) \]

\[ Z[j] = \int [d \rho] W_{\Lambda^{+}}[\rho] \left\{ \frac{\int^{\Lambda^{+}} [d A] \delta(A^{+}) e^{i S_{\Lambda^{+}}[A, \rho]} - \int j \cdot A}{\int^{\Lambda^{+}} [d A] \delta(A^{+}) e^{i S_{\Lambda^{+}}[A, \rho]} \right\} \]

\[ W_{\Lambda^{+}}[\rho] \] Non-pert. gauge invariant “density matrix” defined at initial scale \( \Lambda^{+} \)

For a large nucleus, \( Q_{S}^{2} \propto \mu_{A}^{2} \sim A^{1/3} \Lambda_{QCD}^{2} \); \(\alpha_{S}(Q_{S}^{2}) \ll 1 \) weak coupling EFT!

Simple understanding of “Pomeron” and “Odderon” configurations ...
“Shockwave” propagators

Small fluctuation propagator: $O(\epsilon)$

One loop correction to classical field: $O(\epsilon)$

Effective vertices identical to quark-quark-reggeon and gluon-gluon-reggeon vertices in Lipatov’s Reggeon EFT

$\Gamma_{\mu \nu; \alpha \beta}(p, q) = g_{\mu \nu} \, \frac{1}{4} \, g_{\alpha \beta} - \frac{1}{2} \, g_{\mu \beta} \, g_{\alpha \nu} - \frac{1}{2} \, g_{\mu \alpha} \, g_{\beta \nu} - \frac{1}{2} \, g_{\mu \nu} \, g_{\alpha \beta}$

$T^\mu_{\mu \alpha}(p, q) = -\frac{(2\pi)}{8} \, g_{\mu \alpha} \, T^{\mu \nu}(p, q)$

$U = \text{P} \exp \left( i \int d^4 x \, \left( A^a + A^b \right) \right)$

References:
- Balitsky, Belitsky (2001)
- Iancu, Leonidov, McLerran (2001)
- Bondarenko, Lipatov, Pozdnyakov, Prygarin, arXiv:1708.05183
- Hentschinski, arXiv:1802.06755
Classical lumps in $2 \to N$ scattering and unitarity

False vacuum decay of field configurations $\phi(z)$

Exponential suppression of high occupancy states (classical lumps) unless

$$S \sim \frac{1}{\sqrt{s}} \sim N$$

$$\Rightarrow P_{2\to N} \sim \mathcal{O}(1)$$