"How Difficult is Quantum Many-Body Theory?"

The basic problem of much of condensed matter and high energy physics, as well as quantum chemistry, is to find the ground state properties of some Hamiltonian. Many algorithms have been invented to deal with this problem, each with different strengths and limitations. Ideas such as entanglement entropy from quantum information theory and quantum computing enable us to understand the difficulty of various problems. I will discuss recent results on area laws and use these to prove that we can use matrix product states to efficiently represent ground states for one-dimensional systems with a spectral gap, while certain other one-dimensional problems, without the gap assumption, almost certainly have no efficient way for us to even represent the ground state on a classical computer. I will also discuss recent results on higher-dimensional matrix product states, in an attempt to extend the remarkable success of matrix product algorithms beyond one dimension.