Enhanced Pinning for Vortices in Hyperuniform Substrates and Emergent Hyperuniform Vortex States

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Hyperuniformity – A metric for order


Disordered and hyperuniform

Disordered and not hyperuniform

Variance in number of particles in sampling window grows more slowly than sampling volume

$$\langle N^2_{\Omega} \rangle - \langle N_{\Omega} \rangle^2 \sim R^{d-1}.$$  

$$\sigma_N^2(R) \sim \begin{cases} 
R^{d-1}, & \alpha > 1, \\
R^{d-1} \ln R, & \alpha = 1 \\
R^{d-\alpha}, & 0 < \alpha < 1 
\end{cases}$$  

\((R \to \infty)\)

C.E. Zachary and S. Torquato, PRE 83, 051133 (2011)
Hyperuniformity and the structure factor

For nonhyperuniform disordered patterns, $S(k)$ approaches a finite value as $k \to 0$.

For crystals and other hyperuniform patterns, long wavelength density fluctuations are suppressed:

$$S(k) = \frac{1}{N} |\sum_j \exp(i k \cdot r_j)|^2, (k \neq 0)$$

Disordered hyperuniformity

“Disordered but as uniform as possible”: Chicken cone photoreceptor cells
Y. Jiao, T. Lau, H. Hatzikirou, M. Meyer-Hermann, J.C. Corbo, and S. Torquato,
PRE 89, 022721 (2014)
Nature harnesses hyperuniformity

Experimental images of chicken cone photoreceptor cells

Harnessing hyperuniformity in superconductors

- Superconducting vortices: artificial pinning and critical currents
- Poisson, periodic, and random hyperuniform pinning
- Emergent hyperuniformity of vortices in the pinned state
- Future directions
Superconductor:
• Perfect transport of current
• Complete expulsion of magnetic field.

Vortex: localized magnetic field line
Vortices in Superconductors

• Vortices repel each other and form a triangular lattice.
• Vortex has line energy due to suppression of superconductivity at the vortex core.
• Interaction with defects in the material can produce pinning.

• Pinning (quenched disorder) has always been very poorly characterized.

Critical current: current above which vortices move and dissipate energy
Imaging Techniques for Vortices

Vortex size scale: 100 nm, too small for optical resolution.

Imaging techniques include: Bitter decoration, Hall probe, magnetic force microscopy, magneto-optics; scanning tunnelling spectroscopy, Lorenz microscopy

Bulk measurements: voltage response, magnetization, susceptibility, muon or neutron scattering


Vortex imaging data: late 1960s-present
Statics and Dynamics of Vortices May Have Overlap With Other Systems in Hard and Soft Condensed Matter

Colloids: Ideal system to study a variety of basic problems in condensed matter systems

Colloid Lattice (D Grier, NYU)

Superconducting vortex lattice (JC Davis, Cornell)

Bose-Einstein condensates, Skyrmions in chiral magnets
Artificial Vortex Pinning

- Nanostructuring of superconducting films locally suppresses superconductivity and produces pinning sites
- Good control over pinning geometry and properties has been achieved
- What is best pinning arrangement?

L. van Look, V.V. Moshchalkov (Leuven)
A. Hoffmann, I.K. Schuller (UCSD)
Periodic pinning is known to enhance critical currents

Critical current

Pinning arrays can stabilize different types of vortex crystals and have commensurability effects.

Vortex density: \( B/B_\phi \), where at the matching field \( B_\phi \) the number of vortices equals the number of pins

Kemmler et al, PRL 2006

Harada et al. Science 1996
Easy flow channels reduce critical current away from matching

Square

Triangular

PRB 79, 134501 (2009)

PRL 78, 2648 (1997)

Overdamped equations of vortex motion

\[
\eta \frac{d\mathbf{R}_i}{dt} = F_{i}^{vv} + F_{i}^{p} + F_{i}^{d} + F_{i}^{T}.
\]

\[
F_{i}^{vv} = \sum_{j \neq i}^{N_v} f_0 K_1 \left( \frac{R_{ij}}{\lambda} \right) \mathbf{\hat{R}}_{ij},
\]

\[
f_0 = \phi_0^2 / (2 \pi \mu_0 \lambda^3).
\]

\[
F_{i}^{p} = - \sum_{k=1}^{N_p} \frac{f_p}{R_p} R_{ik}^{(p)} \Theta \left( \frac{R_p - R_{ik}^{(p)}}{\lambda} \right) \mathbf{\hat{R}}_{ik}^{(p)}.
\]

Vortex-vortex interaction: Bessel function

Vortex-pin interaction

Initial vortex positions obtained through simulated annealing. Uniform driving force applied along the positive x direction.
Hyperuniform Pinning Lattice

Unit Cell

Pin placed randomly within this area

Pinning Sites
Two pinning configurations

Disordered hyperuniform

Poisson (Not hyperuniform)

Pin overlap forbidden in each case.
Velocity-force curves

\[ \langle V \rangle = N_v^{-1} \sum v_i \cdot x \]

- **Low field**: \( B/B_\phi = 0.3 \)
- **Matching field**: \( B/B_\phi = 1.0 \)
- **High field**: \( B/B_\phi = 2.7 \)

Red: Hyperuniform
Blue: Poisson
Critical current (depinning force)

R: ratio of hyperuniform to Poisson critical current

Vortex density

F_c vs. B/B_0

R vs. B/B_0
Critical current (depinning force)
Mechanism: fraction of occupied pins

$P_v$: fraction of vortices that sit in pinning sites

Vortex density

Pinning strength

5 unoccupied pins

11 unoccupied pins

Fewer “wasted” pins
S(k) of vortex positions

Pinned state is always hyperuniform!
Pinned vortex state is hyperuniform in each case

Blue: vortices
Orange: pins

Vortex-vortex repulsion imposes energy penalty on local density fluctuations

This could be tested using existing experimental images and/or scattering data

Hyperuniform pinning
Poisson pinning
Proposed static vortex phase diagram

G. Blatter et al, RMP 1994
Dynamic phases of driven vortices

Red: Immobile particles

Blue: Moving particles

Drive is being increased as a function of time

\[ \alpha_m / \alpha_d = 0 \]
Structure Factor $S(k)$ for Driven Vortices

$F_d = 0.0$
Pinned vortex glass

$F_d = 0.5$
Moving liquid

$F_d = 2.0$
Moving liquid

$F_d = 4.0$
Moving smectic
Proposed dynamic phase diagram

Hyperuniform States and Depinning

- Moving Crystal
- Hyper-Liquid
- Random
- Hyper-Pinned

Depinning line

Driving Force vs. Quenched Disorder $\Delta$
Hyperuniform dynamic vortex states already observed?


Bitter decoration measurements

Larger drives: Smectic

Lower drives: “Liquid” (hyperuniform?)
Random Organization and Reversible-Irreversible Transitions: Time to reach steady state after a suddenly applied drive

Dc current suddenly applied (not cycled)

External drive $F_d=0.1$

Cyclically sheared colloid simulation

Plastic flow = irreversible regime
Pinned regime = reversible regime or absorbing state

$V(t) = (V^0 - V^s) \exp(-t/\tau)/t^\alpha + V^s$

PRL 103, 168301 (2009)
Diverging time scale to reach steady state at transition

Our results:

Colloid simulation (Corte et al):
Hyperuniformity in the pinned (absorbed) state but not in the fluctuating (flowing) state?

Reversible state = hyperuniform, even when particle-particle interactions are longer range?
Skyrmions in chiral magnets

Bulk chiral itinerant-electron magnet: MnSi


Magnus Effect on Dynamics

Magnus

Dissipation

Skyrmion

Pinning Site Potential Minima

Vortex

Pinning Site Potential Minima
Drive is being increased as a function of time. Transition into a moving crystal rather than a smectic state due to Magnus term making the dynamic fluctuations isotropic.

Skyrmions: $\alpha_m/\alpha_d=4.0$
Structure Factor $S(k)$ for Driven Skyrmions

$F_d = 0.0$
Pinned skyrmion glass

$F_d = 0.5$
Moving liquid

$F_d = 1.0$
Moving liquid

$F_d = 4.0$
Moving crystal

Skyrmion dynamic phase diagram

Pinned crystal

Moving crystal

Moving liquid

Pinned glass

Hyperuniform?
Conclusions

• Disordered hyperuniform pinning provides critical current enhancement in superconductors by:
  – Eliminating easy-flow channels present in periodic pinning
  – Reducing number of unoccupied pins due to screening

• In the pinned state, vortices adopt a disordered hyperuniform arrangement even when the pinning is non-hyperuniform.
  – Caused by energy penalty for local vortex density fluctuations
  – There is also a flowing hyperuniform state, but no hyperuniformity just above the plastic depinning transition

• Emergent disordered hyperuniform states may have already been observed, but not recognized, for pinned and flowing vortices and colloids, and may exist for skyrmions in chiral magnets.