Simons Collaboration on Confinement and QCD Strings

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Talk at the Inaugural Workshop
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Senior Personnel

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Post-docs include: Francesco Galvagno, Daniel Hackett, Alessandro Nada, Julian Urban, Bernardo Zan
Strong Nuclear Force

• Interactions of particles called hadrons: **baryons** (proton, neutron, Λ, ...) and **mesons** (π, K, ρ, J/psi, ...).

• Short range, ~ 1 fm = 10^{-15} m, but 100 times stronger than electromagnetic.

• Responsible for holding atomic nuclei together.

• Without the Strong Interactions, there would be no world as we know it!
Hadron Substructure

- **Quarks**  
  Gell-Mann, Zweig

- 3 light (u,d,s) and 3 heavy (c,b,t).

- Approximate symmetry relating the light quarks led to grouping of the lightest mesons and baryons into representations of the flavor SU(3).
Quantum Chromodynamics: Celebrating the Golden Jubilee!

- Each quark flavor comes in 3 color states.
  Greenberg; Han, Nambu; Bogoliubov et al.; Bardeen, Fritzsch, Gell-Mann

- The possibility that there is an octet of colored gluons was presented at the ICHEP 1972 Conference, September 6-13. Fritzsch, Gell-Mann

- The SU(3) gauge theory for strong interactions
  Gross and Wilczek; Politzer

\[
\mathcal{L} = \sum_a \bar{\psi}_{q,a} (i \gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} A^C_\mu - m_q \delta_{ab}) \psi_{q,b} - \frac{1}{4} F^{A}_{\mu\nu} F^{A}_{\mu\nu}
\]

\[
F^{A}_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g_s f_{ABC} A^B_\mu A^C_\nu \\
[t^A, t^B] = i f_{ABC} t^C
\]
QCD: a perfect quantum field theory

- Becomes weakly coupled at short distances: Asymptotic Freedom
  Gross & Wilczek; Politzer
- It is the part of the Standard Model that can stand alone: it need not be embedded in a bigger theory.
- Admits a lattice definition that matches onto the Asymptotic Freedom while allowing the theory to be defined non-perturbatively at long distances.
- Lattice calculations of hadron properties approximately match the experiments.
The Confinement Problem

- Why are the colored quarks and gluons not observed as asymptotic states?
- Bound into colorless hadrons.
- On very short time scales, the propagation of colored objects has been observed in a myriad of high-energy collider events.
- Eventually, they hadronize.
- Only colorless bound states are recorded by the detectors.
Clay Mathematics Institute
Millenium Problem # 1

Yang-Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Status: Unsolved
Confining String

• At distances smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
• At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is described by the Nambu-Goto area action with higher derivative corrections.
• Figure based on lattice simulations created by D. Leinweber (Adelaide U.)
Baryons involve string junctions
Large N

• The connection of gauge theory with string theory is strengthened in 't Hooft’s generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).

• Make N large, while keeping the 't Hooft coupling fixed:

\[ \lambda = g_{YM}^2 N \]

• Planar Feynman diagrams. Snapping a flux tube by quark-antiquark creation, or emitting a closed string, is suppressed.
• The string coupling constant \( \sim 1/N \).
• In the large \( N \) limit, we can study the dynamics of a single long confining string. It may connect heavy probe quarks or be wrapped around a large compact direction.

• Talks by S. Dubovsky, A. Athenodorou, M. Caselle, V. Gorbenko, M. Gaberdiel, S. Zare

• Discussion led by O. Aharony

• Talk on high-energy scattering and Pomeron: R. Venugopalan
Understanding Confinement

• In “compact” 3D U(1) gauge theory it is caused by certain topological objects, the monopole-instantons. Polyakov

• In $\mathcal{N}=2$ supersymmetric SU(2) gauge theory broken to $\mathcal{N}=1$, it is due to condensation of monopoles. Seiberg, Witten

• In some supersymmetric models it follows from application of the gauge/string duality. IRK, Strassler; Maldacena, Nunez
• Good control over the dynamics in 1+1 D models for QCD. Talks by G. Tarnopolsky, R. Dempsey, M. Neuzil, M. Yu.

• Yet, no proof has been found for non-Abelian non-supersymmetric gauge theories in 3 and 4 space-time dimensions. This needs to be corrected!

• Talks by V.P. Nair, T. Jacobson, A. Polyakov, J. Kulp.

• Panel on Understanding Confinement: D. Gross, J. Maldacena, N. Seiberg, E. Witten.
• Our goal is to understand the mechanisms of color confinement and UV complete theory of confining strings.

• We anticipate major improvements in Lattice Gauge Theory in the coming few years due to new hardware and Machine Learning algorithms. Talk by P. Shanahan

• For pure gluodynamics, high numerical precision can be achieved.
Schwinger Model

• Quantum Electrodynamics in 1+1 dimensions coupled to a charged fermion of mass \( m \).
  Admits a theta-angle Coleman, Jackiw, Susskind (1975)

\[
\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\theta}{2\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \bar{\Psi} (i\gamma^\mu \partial_\mu - \mathbf{A} - m) \Psi
\]

• Exactly solvable for \( m=0 \) where it reduces to the free Schwinger boson of mass \( e/\sqrt{\pi} \approx 0.56419e \)
• The cases of small and large \( m/e \) may be treated perturbatively.
Lattice Hamiltonian Approach

- Using the staggered fermions Banks, Susskind, Kogut (1975)

\[ H = \frac{e^2 a}{2} \sum_{n=0}^{N-1} \left( L_n + \frac{\theta}{2\pi} \right)^2 + m_{\text{lat}} \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n \]

\[ - \frac{i}{2a} \sum_{n=0}^{N-1} \left[ \chi_n^\dagger U_n \chi_{n+1} - \chi_{n+1}^\dagger U_n^\dagger \chi_n \right] \]


\[ L_n - L_{n-1} = Q_n, \quad Q_n \equiv \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2} \]

- During the past decade, renewed attention due to applications of Density Matrix Renormalization Group (DMRG) Methods. Experimental simulations using cold atoms.
• The lattice approach revisited with the surprising result Dempsey, IRK, Pufu, Zan, arXiv: 2206.05308

\[ m_{\text{lat}} = m - \frac{1}{8} e^2 a \]

• At \( m=0 \) the Hamiltonian is preserved by a “discrete chiral symmetry:” shift by one lattice unit accompanied by \( \theta \to \theta + \pi \)

• The mass shift greatly improves the extrapolation of strong coupling expansions

\[
\begin{align*}
\omega_1 - \omega_0 &= 1 + 2 \mu + \frac{2 x^2}{1 + 2 \mu} - \frac{2 (5 + 2 \mu) x^4}{(1 + 2 \mu)^3} + \frac{4 (59 + 68 \mu + 24 \mu^2 + 4 \mu^3) x^6}{(1 + 2 \mu)^5 (3 + 2 \mu)} \\
\mu &= 2 m_{\text{lat}}/(e^2 a) \\
x^2 &= y = 1/(ea)^4
\end{align*}
\]
In earlier work the massless Schwinger model was assumed to be described by $\mu=0$, and extrapolation to large $x$ did not seem to give good results.

We instead set $\mu = -\frac{1}{4}$ to obtain

$$\delta \omega = \frac{1}{2} + 4y - 72y^2 + 2224y^3 + O(y^4)$$

Pade extrapolating to weak coupling we find

$$E_1 - E_0 \approx \left( \frac{19}{188} \right)^{1/4} e \approx 0.56383e$$

This reproduces the mass of Schwinger boson with error < 0.1 %.
• Exact diagonalizations on lattices with periodic boundary conditions also produced excellent results.

• The DMRG methods, which can give highly precise results, will be reviewed in the talk by G. Tarnopolsky.
Surprises in Hadronic Physics

• Over the past 20 years, many new hadronic states discovered. Their theoretical understanding is far from complete. Talk by T. Skwarnicki

• The narrow charmonium resonance $X(3872)$ is considerably heavier than the $J/\psi (3100)$ whose discovery caused the “November revolution” of 1974 in particle physics.

• Its mass 3872 MeV/c$^2$ is close to the sum of charmed D and D$^*$ meson masses. It may be a molecular state.
• The recent striking discovery by LHCb of the narrow exotic tetraquark state $T_{cc} \ (3875)$ made of $c$-$c$-$\bar{u}$-$\bar{d}$-$\bar{b}$-$\bar{d}$ and other states.

• They pose a challenge for theorists. A more precise understanding of the baryonic string junctions may be important.

• Looking ahead, the Electron-Ion Collider (EIC) is scheduled to begin operation at the Brookhaven National Lab in 9 years.

• It will carry out Proton Tomography with great precision.
Bridging Three Communities

• A special feature of our collaboration will be building bridges connecting the analytical (formal) theory, numerical methods, and hadronic phenomenology communities.

• Interactions between them are currently underutilized. We intend to correct this.
Workshops and Schools

• September 8—10, 2022 – Inaugural workshop at PCTS
• May 2023 – a workshop at the University of Minnesota
• July 10 – 21, 2023 – PITP summer school at IAS, Princeton
• November 2-3, 2023 – the first Annual Collaboration meeting at the Simons Foundation
• March 2024 – workshop at the Weizmann Inst.
• Etc.

• Extend each annual collaboration meeting by 2 workshop days at NYU, Princeton or Long Island
• Additional on-line events: seminars, group meetings, short workshops
Conclusion

• We plan to use an interplay between the analytic approaches, the increasingly precise numerical methods, and inputs from the real world physics, to address the problems in strong interactions with renewed vigor.

• Aim to form a new community proficient in all three.

• Numerical methods likely to speed up greatly in the next few years due to improved algorithms and hardware. They will become a precision tool.

• Ongoing experimental programs at the Large Hadron Collider, Jefferson Lab, Belle II, etc. Run-up to the Electron-Ion Collider at Brookhaven.