Conformal and Near-Conformal Perturbations in Bi-Local Holography

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Overview

- Perturbative (1/N, 1/J) (Witten diagram) expansions in Vectorial Holography: the question of a Bulk AdS description from the Bi-local (collective) scheme.


Ongoing work with: S.Das, A.Gosch, K.Suzuki on: SYK type models: systematics of a Near Conformal expansion and a possible 3D bulk description.
Bi-local Construction

- Bi-local Space (of CFT\_d) : \( \text{AdS}_{d+1} + \text{Spin} (S^{d-1}) \)

\[
\Psi(x_{1}^{\mu}, x_{2}^{\mu}) \Leftrightarrow H(x^{\mu}, z; S)
\]

\[
d + d = (d + 1) + (d - 1)
\]

- Bi-local Field:

\[
\Psi(x_{1}, x_{2}) = \frac{1}{N} \sum_{i=1}^{N} \phi_{i}(x_{1})\phi_{i}(x_{2})
\]
After expanding the bi-local field as

\[ \Psi(x_1, x_2) = \Psi_0(x_1, x_2) + \frac{1}{\sqrt{N}} \eta(x_1, x_2) \]

the action leads to interactions

\[
S[\eta] = \frac{N}{4} \int \prod_{a=1}^{4} d^d x_a \, \eta(x_1, x_2) \mathcal{K}(x_1, x_2; x_3, x_4) \eta(x_3, x_4) \\
- \frac{N}{6} \text{Tr} \left( \Psi_0^{-1} \ast \eta \ast \Psi_0^{-1} \ast \eta \ast \Psi_0^{-1} \ast \eta \right) \\
+ \frac{N}{8} \text{Tr} \left( \Psi_0^{-1} \ast \eta \ast \Psi_0^{-1} \ast \eta \ast \Psi_0^{-1} \ast \eta \ast \Psi_0^{-1} \ast \eta \right) + \cdots
\]
BiLocal Propagator

- **Quartic**
  \[ \hat{L}_{\text{bi}} \approx C_4 + \frac{1}{4} C_2^2 \]
  \[ \approx \frac{1}{4} \left| x_1 - x_2 \right|^4 \left( \frac{\partial}{\partial x_1} \cdot \frac{\partial}{\partial x_1} \right) \left( \frac{\partial}{\partial x_2} \cdot \frac{\partial}{\partial x_2} \right) \]

  \[ C_2 \equiv \frac{1}{2} L_{AB} L^{AB} \]

- **Casimirs**
  \[ C_4 \equiv \frac{1}{4} L_A^B L_B^C L_C^D L_D^A - \frac{1}{2} C_2^2 \]

- **Propagator**
  \[ D(x_1, x_2; x_3, x_4) \propto \sum_{h,s,\vec{p}} \frac{\psi^*_h,s(x_1, x_2; p) \psi_{h,s}(x_3, x_4; p)}{|x_1 - x_2||x_3 - x_4| \lambda_{h,s}} \]
Diagrams

- For tree-level 4-pt func, we have the contact diagram:

- And s-, t- u-channel exchange diagrams:
S-channel Contribution

- Using properties of the 3-vertex, and performing the intermediate integrals, for the s-channel contribution

\[ A_s(1234) = \frac{16}{N} \frac{1}{2} \gamma_\mathcal{O} \frac{1}{|x_{12}| |x_{34}|} G(x_1, x_2; x_3, x_4) \]

\[ = \frac{1}{|x_{12}|^2 |x_{34}|^2} \frac{32}{N} \gamma_\mathcal{O} \sum_{c,s} \frac{2^{5+s}}{(2\pi)^3} \frac{\rho_s(c)}{\lambda_{c,s}} K_{c,s} G_{\frac{3}{2}+c,s}^2(u, v) \]

\[ \lambda_{c,s} = \frac{1}{4} \left[ c^2 - (s - \frac{1}{2})^2 \right] \left[ c^2 - (s + \frac{3}{2})^2 \right] . \]

- The poles: \( c = s - 1/2 \) contributes, the unphysical \( c = s + 3/2 \) do not (zero residue)
4-point (ctnd)

- Further poles showing up in $K$ and $G$
  - Simple poles at $c = s + 2n + \frac{5}{2}$ ($n = 0, 1, 2, \cdots$ and $s = 0, 2, 4, \cdots$) from the factor $\frac{\rho_s(c)}{\lambda_{c,s}} K_{c,s}$
  - Simple poles at $c = s - k + \frac{1}{2}$ ($k = 1, 2, \cdots$, $s$ and $s = 0, 2, 4, \cdots$) from the conformal block $G_{h,s}(u, v)$

- There is a nontrivial cancellation between these additional poles

$$\sum_s \left. \text{Res} \right|_{c=s-\frac{1}{2}} \frac{2^{\frac{5}{2}+s}}{(2\pi)^3} \frac{\rho_s(c)}{\lambda_{c,s}} K_{c,s} G_{\frac{3}{2}+c,s}(u, v)$$

- $s+t+u$ +quartic gives the CFT correlation function.
Question of Locality


- [Sleight & Taronna ’17] [Ponomarev ’17] quartic couplings (and higher) of higher spin theory necessary contain Non-localities.

- BiLocal Construction: Star product: Hamiltonian construction exists Satisfactory Holographic Description
A one (d=1) dimensional model of N-component Majorana fermions with random coupling and nontrivial IR critical point

O(N) symmetry: Singlets: bilocal picture

IR: conformal point: Zero(Goldstone ) mode problem

Perturbation Scheme about the conformal point: Near Conformal Perturbation theory
**SYK MODEL (ctnd)**

- **Partition Function:**
  \[ Z = \int [\mathcal{D}\Psi] \mu[\Psi] e^{-S_{\text{col}}} \]
  
  \[ S_{\text{col}} = \frac{N}{2} \text{Tr}(\partial \star \Psi) + \frac{N}{2} \text{Tr}(\log \Psi) - \frac{J^2 N}{2q} \int d\tau_1 d\tau_2 [\Psi(\tau_1, \tau_2)]^q \]

- **Critical Theory** \((J \to \infty)\) described by \(S_c[\Psi]\)

**Reparametrization Symmetry:**

\[ \Psi(\tau_1, \tau_2) \quad \rightarrow \quad \Psi_f(\tau_1, \tau_2) \equiv \left[ \frac{df(\tau_1)}{d\tau_1} \frac{df(\tau_2)}{d\tau_2} \right]^{\frac{1}{q}} \Psi(f(\tau_1), f(\tau_2)) \]
A. Exact Collective Coordinate Treatment of Reparametrizations

- As a gauge symmetry by Faddeev-Popov:

\[ Z = \int [\mathcal{D}\Psi \mathcal{D}f_\lambda] \mu[\Psi] \det \left( \frac{\delta F[\Psi f]}{\delta f_\lambda} \right) \prod_\lambda \delta [F[\Psi f]] e^{-S_{f,\text{col}}} \]

- Gauge Fixed Representation

\[ Z_f = \int \prod_t df(t) \prod_{t_1,t_2} d\Psi(t_1, t_2) \mu(f, \Psi) \delta \left( \int u_{\frac{3}{2}} \cdot \Psi f \right) e^{-S[f, \Psi]} . \]

- Exclusion of the zero-mode
B. Treatment of Near Conformal Perturb.

- Breaking (of conformal symmetry) comes from:

\[ S[f] = -\frac{N}{2} \int d\tau_1 d\tau_2 \Psi_{0,f}(\tau_1, \tau_2) \delta'(\tau_{12}) \]

- Renormalized source in IR:

\[ \delta'(\tau_{12}) \Rightarrow Q_s(\tau_1, \tau_2) \propto \frac{J^2 \text{sgn}(\tau_{12})}{|J\tau_{12}|^{2-\frac{2}{q}+2s}} + \cdots \]

- Adjusted to vanish on-shell \( s \rightarrow \frac{1}{2} \)

- This leads to the nonlinear Schwarzian Action [AJ & Suzuki ‘16]
Leading Effect (Schwarzian)

\[
S[f] = - \frac{N \alpha}{24\pi J} \int dt \left[ \frac{f'''(t)}{f'(t)} - \frac{3}{2} \left( \frac{f''(t)}{f'(t)} \right)^2 \right]
\]

- Finite result: order of limits \( \text{ir finite, } s=1/2(\text{on shell}) \) uv killed

Similar Evaluation [Kitaev/Suh 17’ ]

Kitaev-Suh source:

\[
s_I(\theta_1, \theta_2) = -a_I \varepsilon^{h_I-1} |\theta_{12}|^{-h_I} \text{sgn}(\theta_{12}) u(\xi)
\]

- With a ‘window ‘function

\[
\int_{-\infty}^{\infty} d\xi \ u(\xi) = 1
\]
Schwarzian - Bilocal Perturbation Scheme

- Interacting vertices come from:
  \[ S_I = \int d\tau_1 d\tau_2 Q_s(\tau_1, \tau_2) \overline{\Psi}_{,f}(\tau_1, \tau_2) \]

- One expands
  \[ \overline{\Psi}_{,f} = |f'(\tau_1)f'(\tau_2)|^{1/q} \overline{\Psi}(f(\tau_1), f(\tau_2)) \]

  with \( f(\tau) = \tau + \epsilon(\tau) \):

  \[ S_I = \int Q_s(\tau_1, \tau_2) \left[ 1 + \epsilon \cdot \hat{d} + \frac{1}{2} \epsilon \epsilon \cdot \hat{V}_3 + \cdots \right] \overline{\Psi}(\tau_1, \tau_2) \]

  where

  \[ \epsilon \cdot \hat{d} = \epsilon(\tau_1) \left( \overrightarrow{\frac{1}{q} \partial_{\tau_1}} - \overleftarrow{\frac{1}{q} \partial_{\tau_1}} \right) + \epsilon(\tau_2) \left( \overrightarrow{\frac{1}{q} \partial_{\tau_2}} - \overleftarrow{\frac{1}{q} \partial_{\tau_2}} \right) \]
Schwarzian – Bi-local Vertices (2)

- We have

\[(Q_s \bar{\Psi})\]  \hspace{1cm}  \(\tau \) \hspace{1cm}  \(\tau_1, \tau_2\)

\[(Q_s \varepsilon \cdot \hat{d} \bar{\Psi})\]  \hspace{1cm}  \(\tau \) \hspace{1cm}  \(\tau_1, \tau_2\)

\[(Q_s \varepsilon \varepsilon \cdot \hat{V}_3 \bar{\Psi})\]  \hspace{1cm}  \(\tau \) \hspace{1cm}  \(\tau_1, \tau_2\)
II: Covariant Observables

- Reparameterization of bi-local fields (change of frame):
  \[ \phi_h(\tau_1, \tau_2) \rightarrow \phi_{h,f}(\tau_1, \tau_2) \equiv |f'(\tau_1)f'(\tau_2)|^h \phi_h(f(\tau_1), f(\tau_2)) \]

- Infinitesimally:
  \[ \delta \phi_h(\tau_1, \tau_2) = \int \frac{d\omega}{2\pi} \varepsilon(\omega) \hat{d}_{h,\omega}(\tau_1, \tau_2) \phi_h(\tau_1, \tau_2) \]
  \[ + \frac{1}{2} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \varepsilon(\omega)\varepsilon(\omega') \hat{V}_{(3);h;\omega,\omega'} \phi_h(\tau_1, \tau_2) + \cdots \]

- This gives a sequence of additional vertices
Two-point Function

- The $1/J$ corrections to the leading $O(J)$ contribution

\[ b_1 = \ \includegraphics[width=0.4\textwidth]{b1.png} \]
\[ b_2 = \ \includegraphics[width=0.4\textwidth]{b2.png} \]
\[ b_3 = \ \includegraphics[width=0.4\textwidth]{b3.png} \]
\[ b_4 = \ \includegraphics[width=0.4\textwidth]{b4.png} \]
Evaluation of $b_1$

- The contribution $b_1$ is written as

$$b_1 = \left\langle \int \frac{d\omega}{2\pi} \varepsilon(\omega) u_{\frac{3}{2},\omega}(\tau_1, \tau_2) \int \frac{d\omega'}{2\pi} \varepsilon(\omega') \hat{d}_{h,\omega'}^x(\tau_3, \tau_4) \int d\tau_5 d\tau_6 D(\tau_5, \tau_6; \tau_3, \tau_4) Q_s(\tau_{56}) \right\rangle$$

Where

$$u_{\frac{3}{2},\omega}(\tau_1, \tau_2) = \hat{d}_{\frac{3}{2},\omega}(\tau_1, \tau_2) \Psi_0(\tau_1, \tau_2)$$

With

$$\lim_{s \to \frac{1}{2}} \int d\tau_3 d\tau_4 D(\tau_1, \tau_2; \tau_3, \tau_4) Q_s(\tau_3, \tau_4) \propto |J\tau_{12}|^{-\frac{2}{q}-1}$$

- This kind of contribution was evaluated as [AJ & Suzuki]

$$b_1 \propto \left( \frac{|z|^{-1} + |z'|^{-1}}{N\tilde{g}'(\frac{3}{2})} \right) \int \frac{d\omega}{\omega^4} u(0),\omega(\tau_1, \tau_2) u^*(0),\omega(\tau_3, \tau_4)$$
Evaluation of $b_4$

- **The contribution $b_4$**

$$b_4 \propto \frac{J^2}{N^2} \lim_{s \to 1/2} \int \frac{d\omega}{|\omega|^4} \int \frac{d\omega'}{|\omega'|^4} u^3_\omega(\tau_1, \tau_2) u^*_\omega(\tau_3, \tau_4)$$

$$\times \int d\tau_5 d\tau_6 d\tau_7 d\tau_8 Q_s(\tau_{56}) Q_s(\tau_{78}) \hat{d}^*_h, -\omega(\tau_5, \tau_6) \hat{d}^*_h, -\omega(\tau_7, \tau_8) \mathcal{D}(\tau_5, \tau_6; \tau_3, \tau_4)$$

- **This was evaluated in (Kitaev & Suh)**

As a nonlocal correction to the Schwarzian

$$S_{\text{non-local}} = - \frac{8}{\pi k'_c(2)} \int dt_1 dt_2 \frac{|f'(t_1)f'(t_2)|^2}{|f(t_1) - f(t_2)|^4} \left[ \ln \frac{|f(t_1) - f(t_2)|^2}{|f'(t_1)f'(t_2)|} + c \right]$$

$$= - \frac{8}{\pi k'_c(2)} \int df_1 df_2 \frac{|f'(t_1)f'(t_2)|}{|f(t_1) - f(t_2)|^4} \left[ \ln \frac{|f(t_1) - f(t_2)|^2}{|f'(t_1)f'(t_2)|} + c \right]$$
Correcting the h=2 mode Propagator

\[
D(t, z; t', z') = (zz')^{\frac{1}{2}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \left[ \frac{3\pi}{2} J_{-\frac{3}{2}}(|\omega|z) J_{-\frac{3}{2}}(|\omega|z') \frac{d}{d\nu} (J_{\nu}(|\omega|z) J_{\nu}(|\omega|z')) \right]_{\nu=\frac{3}{2}} - \frac{2}{\tilde{g}'(\frac{3}{2})} J_{\frac{3}{2}}(|\omega|z) J_{\frac{3}{2}}(|\omega|z') + \frac{3}{2} \frac{J_{\frac{3}{2}}(|\omega|z) J_{\frac{3}{2}}(|\omega|z') \tilde{g}''(\frac{3}{2})}{[\tilde{g}'(\frac{3}{2})]^2} \]

To

\[
\langle \eta(t, z) \eta(t', z') \rangle = (zz')^{\frac{1}{2}} \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-t')} \left[ \frac{2\mathcal{J}}{|\omega|} J_{\frac{3}{2}}(|\omega|z) J_{\frac{3}{2}}(|\omega|z') + \frac{1}{2|\omega|} \left( \frac{1}{z} + \frac{1}{z'} \right) J_{\frac{3}{2}}(|\omega|z) J_{\frac{3}{2}}(|\omega|z') + J_{\frac{3}{2}}(|\omega|z') J_{\frac{3}{2}}(|\omega|z') + J_{-\frac{3}{2}}(|\omega|z) J_{\frac{3}{2}}(|\omega|z') \right] \]

There is: a Gauge where the cancellation is automatic.
SYK as AdS2

- This was the interacting dynamics of the $h=2$ mode: required a near-critical treatment. There is furthermore an infinite sequence of higher states $h>2$ which do not receive corrections.
- They all contribute as intermediate states in 4-point.
- The bi-local representation is not holographic.

\[
S \propto \Psi(t, z) \prod_{m=1}^{\infty} \left( \Box_{\text{AdS}2} - M_m^2 \right) \Phi(t, z),
\]

- With
  \[
  M_m^2 = p_m^2 - \frac{1}{4}, \quad -\frac{2}{3} p_m = \tan \left( \frac{\pi p_m}{2} \right).
  \]

Polchinski & Rosenhaus 16’
SYK as ADS_2?

- Can be understood as coming from a 3rd dimension

\[
\frac{1}{2} \int dt dz dx \sqrt{-g} \left[ -g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - V(x) \Phi^2 \right]
\]

With

\[
 ds^2 = |x|^{4 \rightarrow 1} \left[ \frac{-dt^2 + dz^2}{z^2} + \frac{dx^2}{4|x|(1 - |x|)} \right]
\]

S. Das, A. Ghosh, K. Suzuki, AJ 18'

BiLocal space is too small for a holographic representation

Beyond AdS_2 and BiLocal Holography
The Collective representation can be extended to include correlators of the original (Majorana) Fermions

\[ \langle \chi_{i_1}(t_1) \chi_{i_2}(t_2) \cdots \chi_{i_n}(t_n) \rangle \]

Non-singlet correlation functions : generated by the coupled Lagrangian

\[ S_{com} = S_{coll}(\chi, \Sigma, \Psi) + \int dt_1 \int dt_2 \chi(t_1)\chi(t_2)\Sigma(t_1t_2) \]
Fermions (ctnd)

- The coupling

\[ \chi_i \chi_i \sum \int \chi_i(t_1) \chi_i(t_2) \sum(t_1, t_2) \]

- Through the expansion

\[ \Sigma = \Psi^{-1} = \frac{1}{\Psi_0} - \frac{1}{\Psi_0} \frac{1}{\Psi_0} + \frac{1}{\Psi_0} \frac{1}{\Psi_0} \frac{1}{\Psi_0} + \cdots \]

- Generates the vertices between the (Majorana) fermion and bi-locals \( \bar{\Psi}(t_1 t_2) \)

With the propagator from:

\[ S_2 = \chi(t_1) \psi_0^{-1}(t, t_2) \chi(t_2) \]

And a Rule: Fermions are not to participate in Loops
BULK FERMIONS (in AdS_2)

- It makes sense to have bulk N-component (fermions) coupled to JT gravity
- Relevance of Non-singlets in the bulk have been discussed: c=1/2D String case and BFSS

Study: N-component matter + JT Gravity as an SYK prototype

\[ S_M = -\frac{1}{2} \sum_{i=1}^{N} \int d^2 x \sqrt{-g} \Omega_i(\phi) g^{\mu\nu} \partial_\mu f_i \partial_\nu f_i \]

\[ S_{gr} = \frac{1}{2} \int d^2 x \sqrt{-g} \left[ \phi R + V(\phi) \right] \]
Bulk Fermions (ctnd)

- **Dilaton**
  \[ \Phi = 1 + \frac{a}{z}, \quad a \sim \frac{1}{J} \]

- Almheiri&Polchinski, Almheiri & Kang, Maldacena, Turiaci, Yang: in leading order

- \(1/J\) perturbations: NearConformal perturbation (in progress): adjusting coupling with the Dilaton

\[
S_{\text{reg}} = C_1 \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \: \bar{\chi}_\Delta^{(0)}(\tau_1) \chi_\Delta^{(0)}(\tau_2) \left[ \frac{1}{|\tau_{12}|^{2\Delta}} + \frac{a}{\epsilon} \frac{1}{|\tau_{12}|^{2\Delta}} + \frac{aC_3 \epsilon^{2\Delta - 1}}{C_1} \frac{1}{|\tau_{12}|^{4\Delta}} \right] + \frac{aC_4}{C_1} \frac{1}{|\tau_{12}|^{2\Delta + 1}} \]

Confidence in bulk Fermions.
(Hamiltonian) Picture

- Canonical quantization (with Gauge fixing ) leads to a Hamiltonian for N-component matter +discrete (Schwarzian )

Procedure ( Kuchar, Hajicek, Kunstatter..)

With metric

\[ ds^2 = \Lambda^2 \left[ -\alpha^2 dt^2 + (dz + \beta dt)^2 \right] \]

One has

\[ H \equiv \Pi_\phi \dot{\phi} + \Pi_\Lambda \dot{\Lambda} + \sum_{i=1}^{N} \Pi_i \dot{f}_i - L \]

\[ = \alpha G + \beta F \]
First class Constraints

\[ G = \phi'' - \frac{\Lambda'\phi'}{\Lambda} - \Lambda\Pi_{\Lambda}\Pi_{\phi} - \frac{1}{2}\Lambda^2V(\phi) + \sum_{i=1}^{N} \frac{(\Pi_{f_i})^2}{2\Omega_i} + \sum_{i=1}^{N} \frac{\Omega_i}{2} (f'_i)^2 \]

\[ F = \Pi_{\phi}\phi' - \Lambda\Pi'_{\Lambda} + \sum_{i=1}^{N} \Pi_{f_i}f'_i \]

In a canonical gauge, one can solve the constraints and eliminate the Gravity degrees of freedom (up to a zero mode)
Chang of (Gravitational) Variables

- **Kuchar**

\[ M = \frac{1}{2} \left[ \Pi^2 - \frac{(\phi')^2}{\Lambda^2} + j(\phi) \right], \]

\[ P_M = -\frac{\Lambda \Pi}{2 \left( \Pi^2 - \frac{(\phi')^2}{\Lambda^2} \right)}. \]

- These reduce to discrete modes \( P_M(t, z) = p(t). \) +conjugate.

- One also gauge fixes the Dilaton \( R = \frac{R_0(t)}{z}. \)
Hamiltonian (ctnd)

- And Eliminating its conjugate from the Constraint leads to a nonlinear equation

\[ F' + j' = \left( \frac{2R_0z^2F}{R_0^2 - 4z^4p^2F^2} \right) \sum_{i=1}^{N} \left[ \frac{2z^2pF}{R_0} \Pi^i f_i' + \frac{(\Pi^i_f)^2}{2\Omega_i} + \frac{\Omega_i}{2} (f'_i)^2 \right]. \]

- Solving for \( F(z,t) \) at infinity specifies the canonical Hamiltonian \( H \), for example, when \( p(t) = 0 \)

\[ F(z) = -\exp \left[ -\int_{\epsilon}^{z} d\tilde{z} P(\tilde{z}) \right] \int_{\epsilon}^{z} d\tilde{z} j'(\tilde{z}) \exp \left[ \int_{\epsilon}^{\tilde{z}} d\tilde{\tilde{z}} P(\tilde{\tilde{z}}) \right] + a \exp \left[ \int_{\epsilon}^{z} d\tilde{z} P(\tilde{z}) \right], \]

\[ P(z) = \frac{2z^2}{R_0} \sum_{i=1}^{N} \left[ \frac{(\Pi^i_f)^2}{2\Omega_i} + \frac{\Omega_i}{2} (f'_i)^2 \right]. \]
Transition to 3D

- In canonical quantization, N-component matter is represented through

\[ \Psi(t; z_1, z_2) = \sum_{i=1}^{N} f_i(t, z_1)f_i(t, z_2), \quad \Pi_{\Psi}(t; z_1, z_2) = -i \frac{\delta}{\delta \Psi(t; z_1, z_2)}, \]

a canonical composite field and its conjugate in 3D. The components of the Energy-momentum tensor then become

\[ \mathcal{H}_1(t, z) = \left[ \Pi_{\Psi}(t; z, z') \Psi'(t; z, z') \right]_{z'=z}, \]

\[ \mathcal{H}_2(t, z) = -2 \left[ \Pi_{\Psi} * \Psi * \Pi_{\Psi} \right](t, z) - \frac{1}{2} \left[ \Psi''(t; z, z') \right]_{z'=z}, \]

So that

\[ F' + j' = \left( \frac{2R_0 z^2 F}{R_0^2 - 4z^4 p^2 F^2} \right) \sum_{i=1}^{N} \left[ \frac{2z^2 pF}{R_0} \mathcal{H}_1(t, z) + \mathcal{H}_2(t, z) \right]. \]
Summary

- (Re)analysis of the BiLocal Picture: Holography

We have a Holographic representation: when in corresponding (Witten) diagrams only: Single single traces

SYK: A Higher (dimensional) Collective description is needed for a holographic dual: going beyond bilocals: 3D