Pulsed power driven reconnection and the inverse skin effect

John Greenly, Charles Seyler, Xuan Zhao
Cornell University

With W. Potter, T. Byvank, L. Atoyan

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Pulsed-power driven plasmas want to store up magnetic energy and then pinch together. Can we drive them apart to produce reconnection with outflows?
What happens when the driver voltage reverses and current is driven down at the end of the pulse?
Hannes Alfven discussed inverse skin effects in The Book [1], making the point that these effects are simple in a solid conductor but become quite complex in a movable, fluid conductor like plasma. He wrote:
“A careful analysis of the problem would be very desirable.”
(Cosmical Electrodynamics, p.71)

Malcolm Haines found a simple set of conditions for which he found an analytic solution [2] in 1959.

Jones and Silawatshananai (1980) showed experimentally that inverse skin effects occur in a slow z-pinch plasma.

But I have never heard this phenomenon explicitly invoked in pulsed power-driven plasmas.

So: inverse skin effects

Given a current carried by a bounded conductor, if you drive that current down by reversing the applied electric field at the boundary, on a time scale short compared with the resistive diffusion time, then a reversed current layer forms at the plasma boundary and diffuses into the conductor.

This also reverses the JxB force on that surface layer. In a solid conductor, no problem, but in a plasma that can move, the reversed JxB drives that surface layer outward, away from the body of the plasma (that still carries the original current).
A simple PERSEUS slab simulation illustrates the process:

All plots also show density.
Later in time:

![Graph of magnetic field and ln(density)](image)

![Graph of current in z direction and ln(density)](image)

![Graph of velocity at x direction and ln(density)](image)
Still later: total current is now zero, but internal current and flux remains, with strong outward flow.
• Voltage reversal with a pulsed-power driver drives the current down.

• On time scale short compared with resistive penetration of plasma:

• An “inverse skin effect”
Important: load plasma motion and increasing inductance also produces inverse skin effect. Any dynamics or driving waveform that forces current down faster than field can diffuse out of plasma will do this.

\[ I \frac{dL}{dt} > V \]

\[ \rightarrow B \text{ must decrease} \]
This research uses flows of ablated plasma that were first discovered in MA imploding wire-array Z-pinches.

Wires driven by MA pulsed power break down and form expanding cold plasma “cores”

The cores continuously ablate, feeding a surrounding, hotter “coronal” plasma that carries a large fraction of the current.
If a wire plasma is located in a strong magnetic field gradient, the coronal plasma can be accelerated continuously away from the core to form an ablation stream: Imperial College, Lebedev, Chittenden et al

These streams develop by a complex, diffusive mechanism.

Typical: 100-300 km/s, $10^{17}$/cm$^3$, 30 eV, Al $^{\text{IV}}$

The role of magnetic field in the transition to streaming ablation in wire arrays
M. R. Martin, C. E. Seyler, and J. B. Greenly,
X pinch image at 114 ns
8 mm diameter, 8 wire, W 18µm  COBRA 1843
We are using these ablation flows to study:
→ magnetic reconnection
→ shocks

→ A closely integrated program of experiments on COBRA, and simulation, developing our extended-MHD code, PERSEUS. Problems focused on the plasma-vacuum boundary ubiquitous in pulsed-power driven plasmas.

Relaxation model for extended magnetohydrodynamics:
Comparison to magnetohydrodynamics for dense Z-pinches
C. E. Seyler and M. R. Martin,
Phys. Plasmas 18, 012703 2011
Pulsed-power driven plasmas want to store up magnetic energy and then pinch together. Can we drive them apart to produce reconnection with outflows?
XUV images of a 254 $\mu$m Al wire at 230 ns into a 1 MA COBRA pulse:

End-on view

the wire was here

Side-on view

8 mm

20 mm
What about reconnection?
If you have only one isolated current channel, you don’t need it: magnetic energy can be converted to outward-moving kinetic energy by reversing the driving voltage.

But, in the real world, you almost always have more complicated situations:
A fundamental observation in our experiments so far is the tendency in these systems to form current sheets that expand the X-point magnetic null into an extended null sheet.
This is the basic configuration on the COBRA pulser: 1 MA, 300 ns current pulse:

COBRA coaxial power feed

250mm diameter aluminum wires
Two wire reconnection setup:
Here is the case with two 254 mm Al wires, 16 mm apart. End-on XUV images, 10 ns intervals:

250 µm mylar obstacle
250 µm mylar obstacle
Two 254 mm Al wires, 16 mm separation (vertically in images).

End-on XUV images, 10 ns intervals:
Reversed current in the sheet.
End-on and Side-on XUV images of the same shot show a very narrow central sheet.
Current sheet observations: Laser shadowgraph.
Width of sheet is ~400µm, ion inertial length is ~1mm.
Top view, later time (400 ns), driver current is reversing.
Side view, later time (400 ns).
Laser interferometry (532nm) has given density distributions in the reconnection region.
Two wires

One wire and one 3mm post with plasma inflow on only one side, the current sheet is offset.
Axial interferometry:

The evolution of the reconnection region from an X-point to a long current sheet is shown.

The following sequence of interferograms show times after voltage reversal, and the magnitude of reversed voltage at those times.

No plasma:  

During reconnection:
First measurements of outflow velocity have been made using Doppler shift of visible line radiation. The Al III line at 569.7 nm is intense enough to be seen over the continuum in the two-wire experiments. Using a line of sight along the outflow:

the line is observed by a lens coupled to fiber optic to a streaked spectrometer (.02 nm resolution). The line becomes very broad, with two peaks separated by 0.88 nm, giving velocity of ~240 km/s in each direction during the reconnection outflow. This is highly super-Alfvenic, (~10x) based on $B \sim 5$ T and $n \sim 10^{18}$/ cm$^3$. 
How to diagnose the flows?: Velocity, Mach and Alfven, etc. numbers??
One way: Put in obstacles, study shock structures.
Shock obstacles: vertical strips, 1 mm wide, as tall as the wires

→ Can make inferences about outflow direction and speed
Shock structures vary with distance in the outflow.
XUV images 50,60,70,80 ns after V reversal
Same times, but no V reversal- current crowbarred at peak.
Shocks appear when there is (reversed) electric field.

But not when there is no driving electric field.

(COBRA pulse crowbarred)
The reverse current skin effect layer is repelled from the interior current, and accelerates away the outer mass.

The reversed current is moving outward, so the inductance seen by the driver decreases:
We observe the inductance change as the reversed current moves outward from the wires:
This is what happens when the current does not go down (crowbarred at the end of the pulse):
• Xuan Zhao: DG-PERSEUS studies of magnetized shocks:

• Magnetized plasma flow with field transverse to velocity, incident on a solid obstacle
New work completed in 2014 by PhD student Xuan Zhao:

DG-PERSEUS:

- Built using a much less diffusive discretization method:
- Discontinuous Galerkin method in an exact local div-B-free form.
- able to handle a very wide density range >9 orders of magnitude, not previously implemented with DG
- proven stable with PP limiter.
- solves XMHD on MHD time scale,
- greatly improves accuracy and reduces diffusion compared with FV method. Therefore runs faster- needs fewer cells.
  
    very good for parallel processing with MPI, scaling almost linear.
- easily implements adaptive mesh.

•(1) Our simulation and theory program: the PERSEUS code

•Not:
•code development to match experiments as closely as possible.

•The intent:
•to use the code to help to understand:
  What are the important physical effects in a given situation, and how do they influence the behavior.

•More like “old-fashioned” analytic theory: try a simplified model and see what effects it captures or misses. What matters and what doesn't?

•Specifically: HED plasmas commonly have huge density range: - near-vacuum to solid or above, and very sharp gradients.

•The “vacuum resistivity” problem: MHD tends to put lots of current in low-density regions. Solution: artificial “vacuum resistivity”.

•Charles Seyler found that including electron inertia and Hall terms in the fluid generalized Ohm's law can treat this properly. Seyler and Martin found a computational scheme that made this model run as fast as ordinary MHD.
The resulting code is called PERSEUS:

- Extended-MHD (XMHD) model, a two-fluid model expressed in center-of-mass formulation.
- Uses a local relaxation scheme – (requires Hall term to be included).
- Semi-implicit time advance steps over constraint imposed by stiff source terms.
- Uses a Finite-Volume spatial discretization method.
- (Seyler and Martin, Phys. Plasmas 18, 012703)
Euler Equation with Source:
\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u} + \mathbf{IP}) &= \mathbf{J} \times \mathbf{B} \\
\frac{\partial E_n}{\partial t} + \nabla \cdot (\mathbf{u} (E_n + P)) &= \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J}^2
\end{align*}
\]

Maxwell Equations:
\[
\begin{align*}
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} &= 0 \\
\frac{\partial \mathbf{E}}{\partial t} - c^2 \nabla \times \mathbf{B} &= -\frac{1}{\varepsilon_0} \mathbf{J}
\end{align*}
\]

GOL:
\[
\begin{align*}
\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{uJ} + \mathbf{Ju} - \frac{1}{n_e} \mathbf{JJ}) - \frac{e}{m_e} \mathbf{IP}_e &= \frac{n_e e^2}{m_e} \left[ \mathbf{E} + \mathbf{u} \times \mathbf{B} - \eta \mathbf{J} - \frac{1}{n_e} \mathbf{J} \times \mathbf{B} \right]
\end{align*}
\]

Electron Inertia \quad Electron Pressure \quad Resistive MHD \quad Hall Term

Electron Entropy:
\[
\begin{align*}
\partial_t S_e + \nabla \cdot (u_e S_e) &= (\gamma - 1)n_e^{\gamma-1} - \eta \mathbf{J}^2
\end{align*}
\]
Xuan Zhao: study of shock structures generated by flows past an obstacle.

Figure 7.17: In these three tests, we vary $B$ and $\rho$ at the same time to keep the Mach numbers fixed ($M_s = 16.9$, $M_A = 2.24$, $M_f = 2.22$). Observation: the structures change a lot.
(a) XUV image of two-wire magnetic reconnection experiment with an obstacle in the flow and obstacle outflow stream

(b) Enlargement of the interaction between

(c) Simulation of two-wire magnetic reconnection with an obstacle in the outflow flow and obstacle, contour plot of \( \log(\rho^2 T) \) stream, contour plot of \( \log(\rho) \)

(d) Simulation of the interaction between
New results: Thomson scattering
Laser path
To collection optics
Laser path
A PERSEUS simulation (MHD) during voltage reversal in a similar current pulse: A very thin, elongated central current sheet, with MHD slow shocks.
Flow streamlines, showing redirection across the shocks and continuous acceleration in the outflow.
Magnetic reconnection studies are typically “driven” by moving plasma:

--idealized assumed inflows:
  eg. Sweet-Parker, Petschek

--imposed motion of current channels (flux tubes):
  eg. Solar coronal loop footpoint motion, MRX, Intrator, Swarthmore, etc.

--expanding (thermally driven) plasmas:
  eg. Laser-target plasmas
In this pulsed-power driven regime we control the inductive electric field and rate of change of magnetic energy in the system.  

--the initial accumulation of magnetic and thermal energy in plasma current channels or flux tubes and 

--the final disposition into kinetic energy of outflows by reconnection 

require opposite driving electric fields.
PERSEUS simulations suggest that radiation and opacity are important to the energy accounting in the current sheet, and therefore influence its pressure, thickness, and the reconnection rate.
New studies: 3D reconnection with “guide field” (Bz) made by skewed wires.

Here the skew angle is ~50 degrees: outflow still superalfvenic and supersonic
Skew angle 90 degrees - outflow still supersonic, but not superalfvenic ??
the inverse skin effect: important in other situations?

--Power feed current losses, especially during rapid implosions (large dL/dt)
\[\rightarrow \text{Remember, } (V - I \frac{dL}{dt}) \gg 0 \text{ matters.}\]

--The development of trailing mass and current, and effects on MRT growth in imploding loads.

\[\rightarrow \text{Wire arrays give highest X-ray output when implosion is before peak current (before voltage reversal).}\]
Here’s an example:

A 3 mm diameter solid rod, carrying ~500 kA, with dense, UV- illumination-produced plasma on the right side only. This laser interferogram is ~30 ns after voltage reversal.
Return to the one-wire-plus-post experiment.

The post shows plasma accelerating off the right-hand side after voltage reversal.
PERSEUS simulations of MITL surface plasma. Top pair MHD, bottom pair XMHD (Hall). Poynting flux entering (leaving) from the left before (after) reversal.

These are density maps.

The first of each pair is before voltage reversal, second is after reversal.
532nm interferograms of surface plasma around a 3mm diameter brass post carrying 500kA, and illuminated by radiation from a wire plasma ~10 mm away to the right. First image of each pair is preshot (no plasma), second is ~40ns after voltage reversal. The lefthand pair shows the full length, the righthand pair is an enlargement of a short segment. Surface plasma is seen to accelerate to the right only.
Here is a case where a very basic physics investigation (reconnection) leads us to a new investigation that may have practical implications for pulsed-power experiments.
COBRA 2713 8mm wide, 15 mm tall, <3mm gap, 2x 254
3719: same as 3718
Current and Voltage, 3713

- I-dot
- I
- VLoad
- laser

Current (MA)

Time (ns)

-50 0 50 100 150 200 250 300 350 400

-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5
PERSEUS simulation of MRT growth on an imploding gas puff.

Top: current rises to 100 ns, then constant

Bottom: current rises to 100 ns, then voltage reverses, $\frac{dl}{dt} < 0$ for 50 ns.