Six-Gluon Amplitude at the Origin in Planar N=4 SYM

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LPENS

PCTS Workshop
Large N theories and strings:
conformal, confining, and holographic
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Based on work with
Lance Dixon and Georgios Papathanasiou
Gluon scattering amplitude in planar N=4 SYM

Duality to null Wilson loops:
Efficient use of conformal symmetry

Hexagon function bootstrap:
Powerful way of building amplitudes
at weak coupling

Pentagon flux-tube OPE:
Work at finite coupling

String theory:
Strong coupling description in terms
of minimal surface
Six-gluon amplitude

Because of IR divergences, cut off is needed

Remove divergences in a friendly way

\[ E(u_i) = \lim_{\epsilon \to 0} \frac{\mathcal{A}_6(s_{ij}, \epsilon)}{\mathcal{A}_{6, BDS-like}(s_{ij}, \epsilon)} = \exp \left[ R_6 + \frac{1}{4} \Gamma_{cusp} E^{(1)} \right] \]

with \[ E^{(1)} = \sum_{i=1}^{3} \text{Li}_2(1 - 1/u_i) \]

and with \( \Gamma_{cusp} = \Gamma_{cusp}(g^2) \) the cusp anomalous dimension

What remains is IR finite and function of cross ratios

\[ u_1 = \frac{s_{12}s_{45}}{s_{123}s_{345}}, \quad u_2 = \frac{s_{23}s_{56}}{s_{234}s_{123}}, \quad u_3 = \frac{s_{34}s_{61}}{s_{345}s_{234}} \]
Origin

Triple collinear limit

\[ u_1, u_2, u_3 \to 0 \]
\[ s_{12}, s_{34}, s_{56} \to 0 \]

Kinematics: null triangle; OK for (+ + - -) signature

Observation: Amplitude exhibits Sudakov-like suppression

\[
\log \mathcal{E} = -\frac{\Gamma_{\text{oct}}}{24} \log^2(u_1 u_2 u_3) - \frac{\Gamma_{\text{hex}}}{24} \sum_i \log^2 \left(\frac{u_i}{u_{i+1}}\right) + C_0
\]

up to power suppressed corrections

Amplitude controlled in this regime by a few functions of the coupling only

\[ \Gamma_{\text{oct}} = \Gamma_{\text{oct}}(g^2), \quad \Gamma_{\text{hex}} = \Gamma_{\text{hex}}(g^2), \quad C_0 = C_0(g^2) \]
Weak coupling

Hexagon function bootstrap enabled calculation of six-gluon amplitude through 7 loops throughout the entire kinematical space.

<table>
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<th>$L = 1$</th>
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<tbody>
<tr>
<td>$\Gamma_{\text{oct}}$</td>
<td>4</td>
<td>$-16\zeta_2$</td>
<td>256\zeta_4</td>
<td>$-3264\zeta_6$</td>
<td>$rac{126976}{3}\zeta_8$</td>
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<tr>
<td>$\Gamma_{\text{cusp}}$</td>
<td>4</td>
<td>$-8\zeta_2$</td>
<td>88\zeta_4</td>
<td>$-876\zeta_6 - 32\zeta_3^2$</td>
<td>$rac{28384}{3}\zeta_8 + 128\zeta_2\zeta_3^2 + 640\zeta_3\zeta_5$</td>
</tr>
<tr>
<td>$\Gamma_{\text{hex}}$</td>
<td>4</td>
<td>$-4\zeta_2$</td>
<td>34\zeta_4</td>
<td>$-603\zeta_6 - 24\zeta_3^2$</td>
<td>$rac{18287}{6}\zeta_8 + 48\zeta_2\zeta_3^2 + 480\zeta_3\zeta_5$</td>
</tr>
<tr>
<td>$C_0$</td>
<td>$-3\zeta_2$</td>
<td>$\frac{77}{4}\zeta_4$</td>
<td>$-\frac{4463}{24}\zeta_6 + 2\zeta_3^2$</td>
<td>$\frac{67645}{32}\zeta_8 + 6\zeta_2\zeta_3^2 - 40\zeta_3\zeta_5$</td>
<td>$-\frac{4184281}{160}\zeta_{10} - 65\zeta_4\zeta_3^2 - 120\zeta_2\zeta_3\zeta_5 + 228\zeta_5^2 + 420\zeta_3\zeta_7$</td>
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In particular, through at least 7 loops

$$\Gamma_{\text{oct}} = \frac{2}{\pi^2} \log \cosh (2\pi g) = 4g^2 - \frac{8\pi^2 g^4}{3} + \frac{128\pi^4 g^6}{45} - O(g^8)$$

same as for light-like limit of large charge correlation / octagon

see Gregory’s talk
Collinear limit and flux tube

Collinear limit / OPE

\[ s_{56} \rightarrow 0 \quad u_2 \rightarrow 0 \]

Philosophy: Cusps on opposite sides of square can be thought of as producing excitations propagating from one side to the other. Similar to form factor expansion.

SA/WL as sum over complete basis of intermediate states

\[ \mathcal{W} = \sum_{\psi} C(\psi) e^{-E \tau + i P \sigma + m \varphi} \]

Here \( \tau, \sigma \) and \( \phi \) are parameterizing cross ratios

\[ u_2 \sim e^{-2 \tau} \rightarrow 0 \]
Collinear limit and flux tube

Collinear limit / OPE

$ s_{56} \to 0 \quad u_2 \to 0$

What are the states?

Ground state:
GKP string with energy density (tension) = $\Gamma_{\text{cusp}}$

Excitations: flux-tube fluctuations propagating along the long string

Pretty much like in Sergei’s talk except that here excitations are gapped.
Their energy at rest is measuring the twist of the corresponding adjoint field

(There’s no boost symmetry here on the flux tube world sheet)
Pentagon OPE

Fundamental OPE building blocks are pentagon WLs with insertions

\[ \langle u_1, \ldots, u_N \rangle \]

Pentagon transitions give us control on the OPE series for WLs

\[ \mathcal{W} = \sum_{N=0}^{\infty} \frac{1}{N!} \int \frac{du_1 \ldots du_N}{(2\pi)^N} e^{-\tau \sum_{i=1}^{N} E(u_i)} |P(0|u_1, \ldots, u_N)|^2 \]
Pentagon bootstrap

Bootstrap axioms

I. \( P(u|v) = P(-v|-u) \)

II. \( P(u|v) = S(u,v)P(v|u) \)

III. \( P(u^{-\gamma}|v) = P(v|u) \)

Solution (not unique but very well tested)

\[
P(u|v)^2 = \frac{S(u,v)}{g^2(u-v)(u-v+i)S(u^{-\gamma},v)}
\]

Expressed in terms of flux tube S-matrices
Approaching the origin from OPE

Origin not quite inside the OPE radius of convergency
Problem is similar to the one encountered in Regge theory.
Proceed with Sommerfeld-Watson transform

\[
E_a(p) = a + 2g^2 \left[ \psi(1 + \frac{a + ip}{2}) + \psi(1 + \frac{a - ip}{2}) - 2\psi(1) \right] + O(g^4)
\]

\[
\mathcal{D}_z^{a-1} F_{+z}
\]

Helicity \( a = 1, 2, 3, \ldots \)

Dominant “Reggeon” is found by deforming contour around \( a = 0 \)
Resummed OPE

Massage OPE sum using known pentagon transitions at finite coupling

End result is a matrix-model-like representation for Wilson loop

\[ \mathcal{E} = \mathcal{N} \int \prod_{i=1}^{\infty} d\xi_i^+ d\xi_i^- F(\xi) e^{-\xi \cdot M \cdot \xi} \]

Gaussian dressing factor is controlled by BES kernel

\[ M_{ij}^{mn} = \frac{1}{j} (-1)^{j+1} \delta^{mn} (\delta_{ij} + K_{ij}) \]

with

\[ K_{ij} = 2j(-1)^{ij+j} \int_0^\infty \frac{dt}{t} \frac{J_i(2gt)J_j(2gt)}{e^t - 1} \]
Resummed OPE

Massage OPE sum using known pentagon transitions at finite coupling

End result is a matrix-model-like representation for Wilson loop

\[ \mathcal{E} = \mathcal{N} \int \prod_{i=1}^{\infty} d\xi_i^+ d\xi_i^- F(\xi) e^{-\xi_i \cdot M \cdot \xi_i} \]

Integrand is given by a Fredholm determinant

\[ \log F = - \sum_{N \geq 1} \frac{1}{N} \sum_{a_1, \ldots, a_N} \int \frac{du_1 \ldots du_N}{(2\pi)^N} \prod_{k=1}^{N} \frac{\mu_k e^{\varphi a_k}}{x_k^+ - x_{k+1}^-} e^{2iQ \cdot \xi} \]

Here

\[ x_k^\pm = x(u_k \pm ia_k/2) \]

with Zhukowskii variable

\[ x(u) = \frac{u + \sqrt{u^2 - 4g^2}}{2} \]
Resummed OPE

Massage OPE sum using known pentagon transitions at finite coupling

End result is a matrix-model-like representation for Wilson loop

\[ \mathcal{E} = N \int \prod_{i=1}^{\infty} d\xi_+^i d\xi_-^i \ F(\vec{\xi}) e^{-\vec{\xi} \cdot M \cdot \vec{\xi}} \]

Integrand is given by a Fredholm determinant

\[ \log F = - \sum_{N \geq 1} \frac{1}{N} \sum_{a_1, \ldots, a_N} \left\{ \int \frac{du_1 \cdots du_N}{(2\pi)^N} \prod_{k=1}^{N} \frac{\hat{\mu}_k e^{\varphi a_k}}{x_k^+ - x_{k+1}^-} e^{2i\vec{Q} \cdot \vec{\xi}} \right\} \]

It depends on infinite vector \( \vec{\xi} \) conjugate to family of higher conserved charges

\[ \vec{Q} = \sum_{k=1}^{N} q(u_k, a_k) \]
Simplification

Moment expansion of determinant (tau-function)

$$\log F(\vec{\xi}) = \langle 1 \rangle + 2i \langle Q_i^m \rangle \xi_i^m - 2 \langle Q_i^m Q_j^n \rangle \xi_i^m \xi_j^n + \ldots$$

Origin corresponds to sending $\varphi \to \infty$

All non-Gaussianities are found to vanish in this regime

$$\lim_{\varphi \to \infty} \langle Q_i Q_j Q_k \ldots \rangle = 0$$

Origin is described by quadratic polynomial, as expected

Coefficients are controlled by 0-, 1- and 2-pt functions

$$M_{ij}^{mn} + 2 \langle Q_i^m Q_j^n \rangle$$
In the end: tilted BES kernel

Partition BES matrix kernel into 4 blocks

\[ K = \begin{bmatrix} K_{oo} & K_{o.} \\ K_{.o} & K_{..} \end{bmatrix} \]

Introduce deformation controlled by angle \( \alpha \)

\[ K(\alpha) = 2 \cos \alpha \begin{bmatrix} \cos \alpha K_{oo} & \sin \alpha K_{o.} \\ \sin \alpha K_{.o} & \cos \alpha K_{..} \end{bmatrix} \]

Interesting associated quantities

\[ \Gamma(\alpha) = 4g^2 \left[ \frac{1}{1 + K(\alpha)} \right]_{11} \]

\[ D(\alpha) = \log \det [1 + K(\alpha)] \]
Main observations

Anomalous dimensions controlling origin given in terms of tilted kernels

\[ \Gamma_{\text{oct}} = \Gamma(\alpha = 0) \]
\[ \Gamma_{\text{cusp}} = \Gamma(\alpha = \pi/4) \]
\[ \Gamma_{\text{hex}} = \Gamma(\alpha = \pi/3) \]

Constant is expressed in terms of associated determinants

\[ C_0 = D(\pi/4) - D(\pi/3) - \frac{1}{2}D(0) - \frac{\zeta_2}{2} \Gamma_{\text{cusp}} \]
Easy check at weak coupling

Matrix elements are small at weak coupling \( \mathbf{K}_{ij} = O(g^{i+j}) \)

\[
\frac{1}{1 + \mathbf{K}(\alpha)} = 1 - \mathbf{K}(\alpha) + \mathbf{K}(\alpha)^2 - \ldots
\]

Get agreement with amplitude result through 7 loops

\[
\Gamma(\alpha) = 4g^2 - 16c^2 \zeta_2 g^4 + 32c^2 (3 + 5c^2) \zeta_4 g^6 - 32c^2 ((25 + 42c^2 + 35c^4) \zeta_6 + 4s^2 \zeta_3^2) g^8 + \ldots
\]

Mild dependence on angle \( \alpha \)

Uniform transcendentality
Radius of convergency is independent of angle \( g_c = \pm i/4 \)
Nature of branch point depends on it; e.g. could be

\[
\Gamma(g, \alpha) - \Gamma(g_c, \alpha) \propto (g_c - g)^{2\alpha/\pi}
\]
Special point

Flat angle, the kernel becomes block diagonal

\[
K(0) = \begin{bmatrix} 2K_{oo} & 0 \\ 0 & 2K_{\cdot\cdot} \end{bmatrix}
\]

All odd zeta's go away, left with integer powers of \( \pi^2 \)

The sums can be taken exactly

\[
\Gamma_{\text{oct}} = \Gamma(\alpha = 0) = \frac{2}{\pi^2} \log \cosh (2\pi g)
\]

Similarly for the log of determinant

\[
D(0) = \frac{1}{4} \log \left[ \frac{\sinh (4\pi g)}{4\pi g} \right]
\]

[Belitsky,Korchemsky’19]
Numerics I

Truncate matrix at large order

Anomalous Dimensions $\frac{\Gamma}{2g}$

Solid lines are weak and strong coupling expansion

[Benna, Benvenuti, Klebanov, Scardicchio'06]
Analysis applies to determinant

Log of Determinants: \( \frac{D(\alpha)}{2g} \)

- **finite coupling**
- **oct**
- **cusp**
- **hex**
Strong coupling analysis \[ \sqrt{\lambda} = 4\pi g \to \infty \]

Define

\[ \gamma(t, s) = (J_1(t), iJ_2(t), J_3(t), \ldots) \cdot Q \cdot (1 + K(\alpha))^{-1} \cdot (J_1(s), iJ_2(s), J_3(s), \ldots)^T \]

Problem takes the form of integral equation

\[ \int_{-\infty}^{\infty} dt \, e^{iut} \Omega(t, s)(\cos \alpha + i \sin \alpha \operatorname{sgn} t) = e^{ius} \]

to be solved for \( u^2 < 1 \) and with

\[ \Omega(t, s) = \frac{\cosh \left( \frac{t}{4g} - \alpha \right)}{s \sinh \left( \frac{t}{4g} \right)} \gamma(t, s) \]

Anomalous dimension and determinant follow from

\[ \Gamma(\alpha) = 16g^2 \lim_{s,t \to 0} \frac{\gamma(t,s)}{st} = \frac{4g\Omega(0,0)}{\cos \alpha} \]

\[ \partial_\alpha D(\alpha) = \operatorname{tr} \left[ \frac{\partial_\alpha K(\alpha)}{1 + K(\alpha)} \right] = 2\Re \int_0^\infty dt \frac{ie^{2i\alpha-t/4g} \Omega(t,t)}{\cosh (t/4g - i\alpha)} \]

[Albay, Arutyunov, Benna, Eden, Klebanov'07]
[BB, Korchemsky, Kotanski'07]
[Kostov, Serban, Volin'08]
Strong coupling analysis

Leading order solution has Fourier transform supported on inner interval

\[ \Omega(it, is) = \frac{tV_0(t)V_1(s) - sV_0(s)V_1(t)}{V_1(0)(t - s)} + \ldots \]

with special functions

\[ V_r(t) = \int_{-1}^{1} \frac{du}{2\pi} (1 + u)^{\alpha/\pi - r} (1 - u)^{-\alpha/\pi} e^{iut} \]

It yields the anomalous dimension

\[ \Gamma(\alpha) = \frac{8\alpha g}{\pi \sin (2\alpha)} + O(g^0) \]

and determinant

\[ D(\alpha) = 4\pi g \left[ \frac{1}{4} - \frac{\alpha^2}{\pi^2} \right] - \left[ \frac{1}{4} + \frac{\alpha^2}{\pi^2} \right] \log (4\pi g) + O(g^0) \]
Comments on strong coupling expansion

Possible to systematize analysis for the anomalous dimension

\[ \Gamma(\alpha) = \frac{8\alpha g}{\pi \sin(2\alpha)} \left[ 1 - \frac{s_1}{4\sqrt{\lambda}} - \frac{\alpha s_2}{4\pi \lambda} - \cdots \right] \]

Work out coefficients order by order
(simple generalization of algorithm for cusp anomalous dimension)

Not clear how to do it for determinant yet

Series appears divergent and non-Borel summable, like for cusp anomalous dimension

Nature of “divergence” depends on angle through non-perturbative scale

\[ \Lambda^2 \sim \lambda^{\alpha/\pi} e^{-(1-2\alpha/\pi)\sqrt{\lambda}} \]
Stringy Interlude

String description in terms of surface ending on WL contour at boundary of AdS

Classically, interested in minimal surface area

$$\log \mathcal{W}_6 \approx -\frac{\sqrt{\lambda}}{2\pi} A_6$$

Problem was analyzed and solved in terms of TBA

[Alay,Gaiotto,Maldacena'09]

Hard to solve analytically for generic kinematics

IR regime = collinear limit: iterative solution controlled by fluctuations of GKP string

[Frolov,Tseytlin'02]

UV regime = near origin: harder

[Alday,Gaiotto,Maldacena'09]
[Ito,Satoh,Suzuki'18]
Approaching origin along diagonal

Simplification shows up along the diagonal $u = u_1 = u_2 = u_3$

Exact answer is known in the case and it reproduces the logarithmic scaling when cross ratios are small

$$\frac{1}{2g} \log \mathcal{E}(u, u, u) \sim -\frac{3}{4\pi} \log^2 u$$

String theory prediction $\Gamma_{\text{oct}} \sim 4g/\pi$ in agreement with exact formula

Analysis much harder away from diagonal

Close to origin we can linearize the TBA and replace it by a Fredholm equation

$$f(\theta) = \frac{\varphi - \tau \sqrt{2} \cosh \theta}{\cosh (2\theta)} + \int_{-B}^{B} \frac{d\theta'}{2\pi \cosh (\theta - \theta')} f(\theta')$$

B is Fermi rapidity for sea of lightest charged excitations

[Alday,Gaiotto,Maldacena’09]

[Ito,Satoh,Suzuki’18]
Off diagonal behaviour

Impossible to solve for generic B if not numerically

Suggests origin has a much more complicated structure than at weak coupling

 Might still hope that a matching is possible if we stand close to diagonal

Near diagonal regime maps to large B

Ignoring finite B effects one gets convolution over full axis and equation can be solved by Fourier transform

It yields

$$\Gamma_{\text{hex}} \sim 16g/3\sqrt{3}$$

in agreement with flux-tube prediction

Caveat: finite B effects have no reason to be negligible if we move farther away from diagonal while still staying close to origin; something’s happening at strong coupling....
Stringy Interlude

What is the role of the sphere?

Naively not relevant since not classically excited

\[ \mathcal{W} = \det_{\text{Loops}} \times e^{-2g \times \text{Area}} \]

Effects buried in loop-determinant

Too naive though because the excitations along sphere are becoming gapless…
Constant and beyond minimal area

Fluctuation along sphere produces large corrections

\[ \mathcal{W}_{n=6} = f_6 \lambda^{-\frac{7}{288}} e^{\frac{\sqrt{\lambda}}{144}} - \frac{\sqrt{\lambda}}{2\pi} A_{n=6} \left( 1 + O(1/\sqrt{\lambda}) \right) \]

Quantum pre-factor from O(6) model

\[ f_6 = \frac{1.04}{(\sigma^2 + \tau^2)^{1/72}} + O(e^{-\sqrt{2}\tau}) \]

[BB, Sever, Vieira’14]
Scalar mass is exponentially small at strong coupling

\[ m = \frac{2^{1/4}}{\Gamma(5/4)} \lambda^{1/8} e^{-\frac{\sqrt{\lambda}}{4}} (1 + O(1/\sqrt{\lambda})) \ll 1 \]

Low energy effective theory: (relativistic) O(6) sigma model

\[ \mathcal{L}_{\text{eff}} = \frac{\sqrt{\lambda}}{4\pi} \partial X \cdot \partial X \] with \[ X^2 = \sum_{i=1}^{6} X_i^2 = 1 \]

[Alday,Maldacena'07]
Low energy viewpoint on minimal surface

\[ S_{\text{sphere}} = \frac{\sqrt{\lambda}}{4\pi} \int d^2z \sqrt{g} g^{\alpha\beta} \partial_\alpha X \cdot \partial_\beta X \]

\( S^5 \)

\( \text{AdS}_5 \)

sphere embedding coordinates

induced metric

square

pentagon

hexagon
Low energy viewpoint on minimal surface

Nearly flat everywhere if not for a few points where curvature is concentrated

Pentagon conical defect (adds \( \pi/2 \))

\( S^5 \)

\( \text{AdS}_5 \)

square  pentagon  hexagon
Sphere: 2-pt function of twist operators

\[ \mathcal{W}_6 = \langle 0 | \phi_\square(\tau, \sigma)\phi_\square(0, 0) | 0 \rangle + O(e^{-\sqrt{2}\tau}) \]

O(6) sigma model:

\[ \mathcal{W}_{O(6)}(z) \]

\[ z = m\sqrt{\sigma^2 + \tau^2} \]

Large distance \( z \gg 1 \) \( \mathcal{W}_{O(6)} = 1 + O(e^{-2z}) \)

Short distance \( z \ll 1 \) CFT analysis / power law
CFT analysis

Short distance OPE (valid for $z \ll 1$)

\[ \phi \otimes (\tau, \sigma) \phi \otimes (0, 0) \sim \frac{\log (1/z)^B}{z^A} \phi \otimes (0, 0) \]

Critical exponent $A$

\[ A = 2\Delta \otimes - \Delta \otimes = 2\Delta_{5/4} - \Delta_{3/2} \]

with $\Delta_k$ the scaling dimension of the twist operator $\phi_k$

\[ \Delta_k = \frac{c}{12} (k - \frac{1}{k}) \]

\( c = \text{central charge} \)

\( 2\pi(k - 1) = \text{excess angle for } \phi_k \)
Stringy Interlude

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Effects buried in loop-determinant

Too naive though because the excitations along sphere are becoming gapless…

… quantum shift restore agreement with constant $C_0$ to leading order at strong coupling

\[ C_0/2g \rightarrow -\frac{\pi^2}{12} - \frac{\pi}{6} + \frac{\pi}{72} \]

classical area  quantum sphere
Stringy Interlude

Problem resurfaces in matching prefactor $\lambda^#$

Similar to order of limit issue mentioned by Gregory yesterday

Might come from yet another naively tiny effects…

Sigma model only leading order approximation for sphere

Full thing contains NG action which predicts irrelevant corrections at dimension 4

Effect on dynamics / S-matrix phase is small and power suppressed with energy

\[
S(\theta_1, \theta_2) = S_{O(6)}(\theta_{12}) \times e^{i\delta(\theta_1, \theta_2)}
\]

Same as advocated by Sergei et al. $\delta(\theta_1, \theta_2) = \frac{m^2}{8g} \sinh(\theta_1 - \theta_2) + \ldots$

[BB,Rej'13]
**Stringy Interlude**

Direct effects on pentagon transitions

\[ F(\theta_1, \ldots, \theta_M) = F_{O(6)}(\theta) \times e^{\frac{1}{16g}(E^2 - P^2)} \]

\[ E = m \sum_{i=1}^{M} \cosh \theta_i \quad P = m \sum_{i=1}^{M} \sinh \theta_i \]

Dressed form factor expansion

\[ \mathcal{W}_{\text{scalars}} = \sum_N \int \frac{d\theta_1 \cdots d\theta_N}{(2\pi)^N} e^{-ET - \frac{1}{16g}E^2} F_{O(6)}(\theta_1, \ldots) \]

Origin corresponds to very short distance regime \( T = 0 \)

Gaussian dressing with energy produces a time delay which might fix problem we had
Conclusions

Corner of kinematical space where things simplify

Lots of mysterious relations, starting from the double-logarithmic behaviour itself

Lots of physics involved when comparing with string theory analysis (unfortunately still not completely clear)

Is that specific to six-gluon amplitude? What about higher multiplicity?

Is relation to null large charge correlation a coincidence? or is it hinting at new relations between correlates and WLs?