Traversable Wormholes

Juan Maldacena
Institute for Advanced Study

October, 2018
Introduction/Motivation

• Special relativity is based on the idea of a maximum speed for propagation of signals.

• In general relativity we have a general curved geometry.

• Is there also a maximal speed for propagation of signals?
• In principle, yes. It is given by the light cones in that curved geometry.

• Could we have a curved geometry that allows a ``short cut’’?

There are curved geometries where naively far away points are relatively close by.

• Could quantum fluctuations produce these geometries?
Are they solutions of Einstein equations?
Full Schwarzschild solution

Eddington, Lemaitre, Einstein, Rosen, Finkelstein, Kruskal

Vacuum solution.
Two exteriors, sharing the interior.
Wormhole interpretation.

Figure y: Maximally extended Schwarzschild spacetime. There are two asymptotic regions. The blue spatial slice contains the Einsteint-Rosen bridge connecting the two regions not in causal contact and information cannot be transmitted across the bridge. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nonlocal signal propagation.

Figure z: Another representation of the blue spatial slice of figure y. It contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known, but what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled, we would not connect them by an Einsteint-Rosen bridge. But if they are somehow created at $t=\text{weak entanglement}$, the Neumann representation of the entanglement is as shown in figure z. Of course, in this case, the dynamical decoupling is not.

Non-traversable

No signals

No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar
No science fiction traversable wormholes

• Einstein equations

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} \]

• But the stress tensor has to obey some constraints, it is not totally arbitrary.

• In particular, it is believed it should obey the AANEC.
AANEC

- Achronal Average Null Energy Condition.

\[ \int_{-\infty}^{\infty} dx^- T_{--} \geq 0 \]

- Achronal (fastest line)


- Note: in QFT, we can have \( T_{--} < 0 \) in some regions.
Due to this property of matter, ambient causality is preserved.
But we still have the full Schwarzschild solution to interpret.
Black holes as quantum systems

• A black hole seen from the outside can be described as a quantum system with $S$ degrees of freedom (qubits).
Wormhole and entangled states

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\[
|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R
\]

(in a particular entangled state)

W. Israel
J.M.
J.M. Susskind
• Is it difficult to get two black holes into this state? Or close to this state?

• Is it "stable" in any sense?

• Or can we dismiss these solutions as mathematical curiosities...?

• Can they teach us interesting lessons about quantum gravity?
First analyze this problem in Nearly-AdS$_2$ gravity
Near extremal black holes

\[ M \geq Q \]
\[ M \sim Q \]

It is a critical system!

N-AdS$_2 \times S^2$
The surprisingly simple gravitational dynamics of N-AdS$_2$

NAdS$_2$ = QFT on AdS$_2$ + location of boundary

All gravitational effects

Proper time along the boundary = time of the asymptotically flat region = time of the quantum system

\[(H_L B_{dy} \times H_{bulk} \times H_{R B_{dy}}) / SL(2, R)\]
Add a constant interaction between the dual quantum systems

- Nearly-AdS\(_2\) gravity
- Plus matter
- Plus boundary conditions connecting the two sides

\[
S_{\text{int}} = \mu \int du \chi_L(u)\chi_R(u)
\]

\(u\) is proper length along the boundary, or boundary time.

- This generates negative null energy and allows for an eternally traversable wormhole
NAdS$_2$ gravity + Interaction

Boundaries now move “straight up”

Signals can now propagate from one boundary to the other.
We started with two nearly-critical systems. Added a relevant interaction. Got a system with gap. But whose properties are governed by the properties of the critical system. (eg spectrum of low energy excitations set by the spectrum of anomalous dimensions of the critical model. )
Where does that interaction come from?
Exchange of bulk fields can lead to the interaction between the quantum systems that describe the black hole.
Analogy: Van der Waals interaction

Two neutral atoms exchanging photons.

\[ H_{int} \propto \frac{\vec{d}_L \cdot \vec{d}_R}{d^3} \]

d small enough so that \(1/d\) is larger than the gap between the ground state and the next states.

Entangle the two atoms.
This reasoning inspired the following solution
Wormholes in 4 dimensions, in the Standard Model + gravity
Based on work with:

Alexey Milekhin

Fedor Popov

Related to previous work with Xiaoliang Qi

Inspired by work by Gao, Jafferis and Wall on “Traversable wormholes”
Drawing by John Wheeler, 1966

Charge without charge.  Spatial geometry. Traversable wormhole
Mass without mass
Recall a classic result
It is a Long wormhole

- It takes longer to go through the wormhole than through the ambient space.
- Not possible in classical physics due to the Null Energy Condition. Because the NEC is true.
- Need quantum effects to violate the ANEC. Casimir energy.

- Can we do it in a controllable way?
Negative null energy in QFT

Eg. Two spacetime dimensions

\[ T_{++} < 0 \]
\[ E \propto -\frac{1}{L} \]

The null energy condition does not hold for null lines that are not achronal!
Some necessary elements

• We need something looking like a circle to have negative Casimir energy.

• Large number of bulk fields to enhance the size of quantum effects.

• We will show how to assemble these elements in a few steps.
The theory

\[ S = \int d^4 x \left[ M_{pl}^2 R - \frac{1}{g^2} F^2 + i \bar{\psi} \slashed{D} \psi \right] \]

Einstein + U(1) gauge field + massless charged fermion

Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).
The first solution: Extremal black hole

Magnetic charge $q$

$$\int_{S^2} F = q = \text{integer}$$

$$r_e \sim q$$

$AdS_2 \times S^2$

$l_{\text{Planck}} = 1$
The next solution: Near Extremal black hole

\[ M = r_e + r_e^3 T^2 = r_e + \frac{r_e^3}{\beta^2} \]

\[ r_e \sim q \]

Very small

\[ l_{\text{Planck}} = 1 \]

\[ \beta \] is the "length" of the throat. Redshift factor between the top and the bottom.
Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.

Massless fermions $\rightarrow$ U(1) chiral symmetry

4d anomaly $\rightarrow$ 2d anomaly $\rightarrow$ there should be massless fermions in 2d.

(Here we view F as non-dynamical).
Motion of charged fermions

• Degeneracy = q = flux of the magnetic field on the sphere. Form a spin $j$, representation of SU(2), $2j + 1 = q$.

• We effectively get q massless two dimensional fermions along the time and radial direction.

• We can think of each of them as following a magnetic field line.
massless two dimensional fields, along field lines.
$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$

Global

$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{(r^2 - 1)}$

Thermal/Rindler

$\text{AdS}_2$
Nearly AdS$_2$

\[ ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma} \]

Global

\[ ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{(r^2 - 1)} \]

Thermal/Rindler

Connect them to flat space, so that $t$ is an isometry. They acquire non-zero energy when the throat has finite length

\[ M = r_e + r_e^3 T^2 = r_e + \frac{r_e^3}{\beta^2} \]
Connect a pair black holes

connect and in global $\text{AdS}_2$
We describe the solution by joining three approximate solutions. Joined via overlapping regions of validity.

One has positive magnetic charge, the other negative.
Fermion trajectories

Charged fermion moves along the magnetic field lines. Closed loop.

Positive magnetic charge

Negative magnetic charge
Casimir energy

Assume: “Length of the throat” is larger than the distance.

\( L = \) time it takes to go through the throat as measured from outside

\( L \gg d \)

Casimir energy is of the order of

\[
E \propto -\frac{q}{L+d} \sim -\frac{q}{L}
\]

Full energy also needs to take into account the conformal anomaly because \( \text{AdS}_2 \) has a warp factor.
That just changes the numerical factor.

\[
L \gg L_{\text{out}} > d
\]
Finding the solution

Solve Einstein equations in the throat region with the negative quantum stress tensor

Balance the classical curvature + gauge field energy vs the Casimir energy.

\[ M - q = \frac{q^3}{L^2} - \frac{q}{L}, \quad \frac{\partial M}{\partial L} = 0 \rightarrow L \sim q^2 \]

Now the throat is stabilized. Negative binding energy.

\[ E_{\text{binding}} = M - q = -\frac{1}{q} = -\frac{1}{r_s} \]

Very small. Only low energy waves can explore it
This is not yet a solution in the outside region:

The two objects attract and would fall on to each other
Adding rotation

\[ d \gg r_e \]

\[ \Omega = \sqrt{\frac{r_e}{d^3}} \]

Kepler rotation frequency
Throat is fragile

• Rotation $\rightarrow$ radiation $\rightarrow$ effective temperature: $T = \Omega$

$$\Omega = \sqrt{\frac{r_e}{d^3}}$$  Kepler rotation frequency

• We need that $\Omega$ is smaller than the energy gap of the throat

$$\Omega \ll \frac{1}{L}$$

• The configuration will only live for some time, until the black holes get closer..

• These issues could be avoided by going to $\text{AdS}_4$...
Some necessary inequalities

\[ L \sim q^2 \]  
From stabilized throat solution

\[ d \ll L \rightarrow d \ll q^2 \]  
Black holes close enough to that Casimir energy computation was correct.

\[ \sqrt{\frac{q}{d^3}} = \Omega \ll \frac{1}{L} \rightarrow q^{\frac{5}{3}} \ll d \]  
Black holes far enough so that they rotate slowly compared to the energy gap.

Kepler rotation frequency

Unruh-like temperature less than energy gap

They are compatible

\[ q^{\frac{5}{3}} \ll d \ll q^2 \]

Other effects we could think off are also small:
can allow small eccentricity, add electromagnetic and gravitational radiation, etc. Has a finite lifetime.
Length scales

\[ r_s \sim q \ll d \ll L \sim q^2 \ll \Omega^{-1} \]

- Size of each black hole and inverse of binding energy of the wormhole
- Distance between black holes
- "Length" of throat or time it takes to go through the wormhole.
- Inverse rotation frequency
- Inverse energy gap. Redshifted energy for excitations deep in the wormhole.
Final solution

Looks like the exterior of two near extremal black holes. But they connected. But there is no horizon!. Zero entropy solution. It has a small binding energy.
Entropy and entanglement

- Total spacetime has no entropy and no horizon.
- If we only look at one object $\rightarrow$ entanglement entropy $=$ extremal black hole entropy
- Wormhole $=$ two entangled black holes
- Total Hamiltonian $\quad H = H_L + H_R + H_{\text{int}}$

Generated by fermions in exterior
• If we were to disconnect the black holes in the exterior → the interior would evolve into the geometry of the eternal near-extremal black hole.
Rotation $\Rightarrow$ temperature

$$T \sim \Omega \ll E_{\text{gap}} \sim \frac{1}{L}$$

Temperature does not create particles in the throat

Two Black Holes: $F = -TQ^2$

Wormhole: $F = -E_{\text{binding}} = -\frac{1}{Q}$

Wormhole is the stable thermodynamic phase for $T < 1/Q^3$

For the solution we described so far: Wormhole is metastable.
Length $L$ as $d$ increases

$$E_c \propto \frac{1}{4L} + \frac{1}{L + d}$$

Conformal anomaly

Casimir cylinder

Stops being classical
Wormholes in the Standard Model

If nature is described by the Standard Model at short distances and $d$ is smaller than the electroweak scale,

$$1 \ll q \ll 10^8$$

Distance $d$ smaller than electroweak scale.

If the standard model is not valid $\rightarrow$ similar ingredients might be present in the true theory.

That it can exist, does not mean that it is easily produced by some natural or artificial process.
They are connected through a wormhole!

Much smaller than the ones LIGO or the LHC can detect!

Pair of entangled black holes.
Conclusions

• We displayed a solution of an Einstein Maxwell theory with charged fermions.
• It is a traversable wormhole in four dimensions and with no exotic matter.
• It balances classical and quantum effects.
• It has a non-trivial spacetime topology, which is forbidden in the classical theory.
• Can be viewed as a pair of entangled black holes.
• But it has no horizon and no entropy.
Questions

• If we start from disconnected near extremal black holes: Can they be connected quickly enough? → topology change.