Current Interactions from Nonlinear Higher-Spin Equations and Holography

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HS holography

General idea of HS duality Sundborg (2001), Witten (2001)

$AdS_4/CFT3$: Klebanov, Polyakov (2002); Giombi, Yin (2009,2010)

Aharony, Gur-Ari, Yacoby (2011) extension to duality between CS boundary theory and HS theories in AdS_4 with various η.

Explicit check was only partially successful Giombi and Yin (2012)

Standard computations with the homotopy operator ∂_Z does not lead to the local frame in HS theory. We announce the result of application of an alternative homotopy operator $\mathcal{P}_{HS}$: leading to local cubic coupling with the coupling constant $\eta\bar{\eta}$ independent of the phase of η.

- Independence on the phase of η leads to anticipated boundary results
- Is in agreement with the conjecture on self-dual HS theory

Main result: computation of the explicit form of local current interactions in AdS_4 HS theory

Frame-like formulation of HS fields

$g_{nm} \longrightarrow h_n^{\alpha\dot\alpha} \longrightarrow \{h_n^{\alpha\dot\alpha}, \omega_n^{\alpha\beta}, \bar\omega_n^{\dot\alpha\dot\beta}\} \qquad \alpha, \dot\alpha = 1, 2$

admits a natural generalization to $s \geq 2$

$\varphi_{n_1 \dots n_s} \to h_n{}^{\alpha_1 \dots \alpha_{s-1}\, \dot\alpha_1 \dots \dot\alpha_{s-1}} \to \qquad \omega_{\alpha_1 \dots \alpha_n\, , \dot\beta_1 \dots \dot\beta_m}\,, \qquad n + m = 2(s-1)$

$s = 1: \quad \omega(x) = dx^n \omega_n(x)$

$s = 2: \quad \omega_{\alpha\dot\beta}(x)\,, \qquad \omega_{\alpha\beta}(x)\,, \qquad \bar\omega_{\dot\alpha\dot\beta}(x)$

$s = 3/2: \quad \omega_\alpha(x)\,, \qquad \bar\omega_{\dot\alpha}(x)$

Frame-like fields: $|n - m| = 0$ **(bosons) or** $|n - m| = 1$ **fermions**

By virtue of constraints $t = [\frac{1}{2}|n - m|]$ **is an order of derivatives**

$$\omega_{\alpha_1 \dots \alpha_n\, , \dot\alpha_1 \dots \dot\alpha_m} = \Pi \Big(\partial^t h_{\alpha_1 \dots \alpha_{n_0}\, \dot\alpha_1 \dots \dot\alpha_{m_0}} \Big)\,, \qquad |n_0 - m_0| < 2$$

First-order unfolded equations: spin two example

Fields: $h^{\alpha\dot\alpha}$ **and** $\omega^{\alpha\beta}$, $\bar\omega^{\dot\alpha\dot\beta}$. **Zero-torsion constraint and Einstein equations**

$$R_{\alpha\dot\alpha} = 0\,, \qquad R_{\alpha_1\alpha_2} = h^{\alpha_3}{}_{\dot\alpha} \wedge h^{\alpha_4\dot\alpha} C_{\alpha_1\ldots\alpha_4}\,, \qquad \overline{R}_{\dot\alpha_1\dot\alpha_2} = h_{\alpha}{}^{\dot\alpha_3} \wedge h^{\alpha\dot\alpha_4}\overline{C}_{\dot\alpha_1\ldots\dot\alpha_4}$$

$C_{\alpha_1\ldots\alpha_4}$ **and** $\overline{C}_{\dot\alpha_1\ldots\dot\alpha_4}$: **Weyl tensor**

Bianchi identities + Einstein equations imply

$$D^L C_{\alpha_1\ldots\alpha_{n+4}\,,\dot\alpha_1\ldots\dot\alpha_n} + h^{\alpha_{n+5}\dot\alpha_{n+1}} C_{\alpha_1\ldots\alpha_{n+5}\,,\dot\alpha_1\ldots\dot\alpha_{n+1}} = O(C^2)$$

$$D^L \overline{C}_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_{n+4}} + h^{\alpha_{n+1}\dot\alpha_{n+5}} \overline{C}_{\alpha_1\ldots\alpha_{n+1}\,,\dot\alpha_1\ldots\dot\alpha_{n+5}} = O(C^2)$$

$C_{\alpha_1\ldots\alpha_{n+4}\,,\dot\alpha_1\ldots\dot\alpha_n}$ **and** $\overline{C}_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_{n+4}}$: **order-$n$ on-shell nontrivial derivatives of the Weyl tensor**

Analogously, for general-spin 0-forms $\qquad C_{\alpha_1\ldots\alpha_n\,,\dot\beta_1\ldots\dot\beta_m}\,, \qquad |n-m| = 2s$

$s = 0: \quad C(x)$

$s = 1/2: \quad C_\alpha(x)\,, \qquad \bar C_{\dot\alpha}(x)$

$s = 1: \quad C_{\alpha\beta}\,, \qquad \bar C_{\dot\alpha\dot\beta}$

$s = 3/2: \quad C_{\alpha\beta\gamma}\,, \qquad \bar C_{\dot\alpha\dot\beta\dot\gamma}$

$s = 2: \quad C_{\alpha_1\ldots\alpha_4}\,, \qquad \bar C_{\dot\alpha_1\ldots\dot\alpha_4}$

Free Massless Fields of all Spins: Central On-shell theorem

Infinite set of integer spins

$\omega(y,\bar y\mid x)\,,\quad C(y,\bar y\mid x)\quad f(y,\bar y)=\sum_{n,m=0}^{\infty}\frac{1}{n!m!}f_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}y^{\alpha_1}\dots y^{\alpha_n}\bar y^{\dot\alpha_1}\dots\bar y^{\dot\alpha_m}$

Fermions require doubling of fields

The full unfolded system for free fields is

$$\star\qquad R_1(y,\bar y\mid x)=\frac{i}{4}\left(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar y^{\dot\alpha}\partial\bar y^{\dot\beta}}\;C(0,\bar y\mid x)+\bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\;C(y,0\mid x)\right)$$

$$\star\qquad \tilde D_0 C(y,\bar y\mid x)=0$$

Vacuum: $\quad R_{\alpha\beta}=0\,,\quad \overline{R}_{\dot\alpha\dot\beta}=0\,,\quad R_{\alpha\dot\alpha}=0\;:\qquad sp(4)\sim o(3,2)$

$$R_{\alpha\beta}:=d\omega_{\alpha\beta}+\omega_{\alpha\gamma}\omega_{\beta}{}^{\gamma}-\;H_{\alpha\beta}\,,\qquad R_{\alpha\dot\beta}:=d+\omega_{\alpha\gamma}h^{\gamma}{}_{\dot\beta}+\overline{\omega}_{\dot\beta\dot\delta}h_{\alpha}{}^{\dot\delta}$$

$$R_1(y,\bar y\mid x)=D_0^{ad}\omega(y,\bar y\mid x)\qquad H^{\alpha\beta}=h^{\alpha}{}_{\dot\alpha}\wedge h^{\beta\dot\alpha}\,,\quad \overline{H}^{\dot\alpha\dot\beta}=h_{\alpha}{}^{\dot\alpha}\wedge h^{\alpha\dot\beta}$$

$$D_0^{ad}\omega=D^L-h^{\alpha\dot\beta}\left(y_\alpha\frac{\partial}{\partial\bar y^{\dot\beta}}+\frac{\partial}{\partial y^{\alpha}}\bar y_{\dot\beta}\right)\,,\qquad \tilde D_0=D^L+h^{\alpha\dot\beta}\left(y_\alpha\bar y_{\dot\beta}+\frac{\partial^2}{\partial y^{\alpha}\partial\bar y^{\dot\beta}}\right)$$

$$D^L A=d_x-\left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^{\beta}}+\bar\omega^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\frac{\partial}{\partial\bar y^{\dot\beta}}\right)$$

Self-duality

Parameters η and $\bar\eta$ are complex conjugated

$\bar\eta\, C(y,0\mid x)$ and $\eta\, C(0,\bar y\mid x)$ are self-dual and anti-self-dual components of the analogues of the Weyl tensor for all massless fields

At $(\eta)\bar\eta = 0$ the free theory is (anti-)self-dual.

$C(y,0\mid x)$ and $C(0,\bar y\mid x)$ are primaries of the conformal modules. Along with descendants the (anti-)self-dual conformal modules are described by $C(y,\bar y\mid x)$ with (negative)positive helicities h

$$(y^\alpha \frac{\partial}{\partial y^\alpha} - \bar y^{\dot\alpha}\frac{\partial}{\partial \bar y^{\dot\alpha}})C(y,\bar y\mid x) = 2hC(y,\bar y\mid x)$$

By virtue of the equation $\tilde D_0 C(y,\bar y\mid x) = 0$ higher polynomials in y, $\bar y$ describe higher space-time derivatives of the primary fields

Fields of the Nonlinear System

Closed formulation of nonlinear equations demands the doubling of spinors and Klein operator

$$\omega(Y|x) \longrightarrow W(Z;Y|x)\,, \qquad C(Y,k|x) \longrightarrow B(Z;Y;k|x)$$

Some of the nonlinear HS equations determine the dependence on the additional variables Z_A in terms of "initial data"

$\omega(Y|x) := W(0;Y|x)$

$C(Y;k|x) := B(0;Y;k|x)$

$S(Z,Y|x) = dZ^A S_A(Z,Y|x)$ is a connection along Z^A

Klein operator k generates chirality automorphisms

$$kf(A) = f(\tilde{A})k\,, \quad A = (a_\alpha\,, \bar{a}_{\dot{\alpha}}): \quad \tilde{A} = A = (-a_\alpha\,, \bar{a}_{\dot{\alpha}})$$

$$P(Y) = P^{\alpha\dot{\alpha}} y_\alpha \bar{y}_{\dot{\alpha}} \quad \longrightarrow \quad \tilde{P}(Y) = -P(Y)\,, \qquad \tilde{M}(Y) = M(Y)$$

Nonlinear HS Equations

HS star product

$$(f \star g)(Z,Y) = \int dS dT f(Z+S, Y+S) g(Z-T, Y+T) \exp -iS_A T^A$$

$$[Y_A, Y_B]_\star = -[Z_A, Z_B]_\star = 2iC_{AB}\,, \qquad Z-Y \,:\, Z+Y \text{ normal ordering}$$

Inner Klein operators:

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \bar\kappa = \exp iz_{\dot\alpha} y^{\dot\alpha}\,, \qquad \kappa \star f = \tilde f \star \kappa\,, \qquad \kappa \star \kappa = 1$$

$$\begin{cases} dW + W \star W = 0 \\ dB + W \star B - B \star W = 0 \\ dS + W \star S + S \star W = 0 \\ S \star B - B \star S = 0 \\ S \star S = i(dZ^A dZ_A + dz^\alpha dz_\alpha F_\star(B) \star k \star \kappa + d\bar z^{\dot\alpha} d\bar z_{\dot\alpha} \bar F_\star(B) \star k \star \bar\kappa) \end{cases}$$

1992

$$F_\star(B) = \sum_{n=1}^{\infty} f_n \underbrace{B \star B \ldots \star B}_{n}\,, \qquad \bar F_\star(B) = \sum_{n=1}^{\infty} \bar f_n \underbrace{B \star B \ldots \star B}_{n}$$

Self-dual HS equatione where conjectured to be described by the nonlinear HS equations with $F_\star(B) = 0$

Perturbative analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = dZ^A Z_A\,, \qquad W_0 = \frac{1}{2}\omega_0^{AB}(x) Y_A Y_B$$

$$dW_0 + W_0 \star W_0 = 0$$

$\omega_0^{AB}(x)$: **describes** AdS_4.

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

$$[S_0\,, f]_\star = -2i d_Z f\,, \qquad d_Z = dZ^A \frac{\partial}{\partial Z^A}$$

Reconstruction of Z^A variables

Perturbatively, equations containing S have the form

$$d_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|dZ) \qquad d_Z V[U_{<n}](Z;Y|dZ) = 0$$

they can be solved as

$$U_n(Z;Y|dZ) = \partial_Z V[U_{<n}](Z;Y|dZ) + h(Y) + d_Z\epsilon(Z;Y|dZ)$$

For instance

$$\partial_Z V(Z;Y|dZ) = \int_0^1 \frac{dt}{t} V(tZ;Y|tdZ)$$

Alternative forms of ∂_Z that differ by d_Z-closed forms can also be used.

Nontrivial space-time equations on $\omega(Y|x)$ and $C(Y|x)$ are in the sector of dZ-cohomology

Central On-Shell Theorem is reproduced in the lowest order with

$\eta = f_1, \quad \bar{\eta} = \bar{f}_1$

Conserved currents and current deformation

Conserved currents $J(Y_1, Y_2|x)$ are associated with the bilinears of $C(Y|x)$

$$J(Y_1, Y_2|x) := C(Y_1|x)\tilde{C}(Y_2|x)\,, \qquad \tilde{C}(y,\bar{y}|x) = C(-y,\bar{y}|x)\,.$$

As a consequence of the rank-one equation for $C(Y|x)$,

$J(Y_1, Y_2|x)$ obeys the current equation Gelfond, MV (2003)

$$\tilde{D}_2 J(Y_1, Y_2|x) = 0\,, \qquad \tilde{D}_2 := D^L - i\lambda h^{\alpha\dot{\beta}}\Big(y_{1\alpha}\bar{y}_{1\dot{\beta}} - y_{2\alpha}\bar{y}_{2\dot{\beta}} - \frac{\partial^2}{\partial y_1^{\alpha}\partial \bar{y_1}^{\dot{\beta}}} + \frac{\partial^2}{\partial y_2^{\alpha}\partial \bar{y_2}^{\dot{\beta}}}\Big)$$

Current deformation have a form of a linear system

$$D\omega - L(w,C) + \Gamma(w,J) = 0\,,$$

$$\tilde{D}C + \mathcal{H}(w,J) = 0\,, \qquad \tilde{D}_2 J(Y_1, Y_2|x) = 0$$

$$L(w,C) := \frac{i}{4}\Big(\eta \overline{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \overline{y}^{\dot{\alpha}} \partial \overline{y}^{\dot{\beta}}}\ \bar{C}(0,\overline{y}|x) + \bar{\eta} H^{\alpha\beta} \frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0|x)\Big)$$

Linear functionals Γ and $\mathcal{H}$ should obey the compatibility conditions

Nonlocality of HS Gauge Theory

Having infinitely many HS fields with higher derivatives in interactions, the HS Gauge Theory is not local

$\lambda^{-1}D \sim 1$ since $[\lambda^{-1}D\,,\lambda^{-1}D] \sim 1$

A different mass parameter like α' is needed for a low-energy expansion

Technically, locality is due to the absence of integration over s and t.

$$\int \frac{dsdt}{(2\pi)^2}\exp i[s_\beta t^\beta] f(y+s,\bar{y})g(y+t,\bar{y}) = f(y,\bar{y})\exp[-i\overleftarrow{\partial}_\alpha \overrightarrow{\partial}_\beta \epsilon^{\alpha\beta}]g(y,\bar{y})$$

$$\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2}\exp i[\bar{s}_{\dot\beta} \bar{t}^{\dot\beta}] f(y,\bar{y}+\bar{s})g(y,\bar{y}+\bar{t}) = f(y,\bar{y})\exp[-i\overleftarrow{\partial}_{\dot\alpha} \overrightarrow{\partial}_{\dot\beta} \epsilon^{\dot\alpha\dot\beta}]g(y,\bar{y})$$

For given helicities carried by g and f, only a single term in the sum contributes hence containing a finite number of derivatives.

When both integrations are present, the number of derivatives in y and $\bar{y}$ can infinitely increase without affecting the helicities carried by g and f, implying appearance of infinite tails of derivatives and hence nonlocality.

Current deformation from nonlinear equations

In the 0-form sector the deformation is

$$D_0 C + [\omega\,, C]_\star + \mathcal{H}(w, J) = 0\,,$$

$$J(y_1, y_2; \bar{y}_1, \bar{y}_2|x) = C(y_1, \bar{y}_1|x)\tilde{C}(y_2, \bar{y}_2|x)$$

A simple computation using the canonical homotopy operator ∂ gives

$$\mathcal{H}(w, J) = \mathcal{H}_\eta(w, J) + \mathcal{H}_{\bar{\eta}}(w, J)\,,$$

$$\begin{aligned}\mathcal{H}_\eta(w, J) = & -\frac{i}{2}\eta \int \frac{dSdT}{(2\pi)^4} \exp iS_A T^A \int_0^1 d\tau \\ & [h(s, \tau\bar{y} - (1-\tau)\bar{t})J(\tau s, -(1-\tau)y + t; \bar{y} + \bar{s}, \bar{y} + \bar{t}) \\ & -h(t, \tau\bar{y} - (1-\tau)\bar{s})J((1-\tau)y + s, \tau t, \bar{y} + \bar{s}; \bar{y} + \bar{t})]\end{aligned}$$

$$h(u, \bar{v}) = h^{\alpha\dot{\beta}} u_\alpha \bar{v}_{\dot{\beta}}$$

The result is not local containing integrations over both s, t and $\bar{s}, \bar{t}$.

Field redefinition

Alternative HS homotopy operator ∂_{HS} gives

$$\mathcal{H}_{\eta\, cur}(w,J) = \frac{\eta}{4}\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2}\exp i[\bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}]\int_0^1 d\tau h(y,\tau\bar{s}+(1-\tau)\bar{t}) \\ J(\tau y,(\tau-1)y;\bar{y}+\bar{s},\bar{y}+\bar{t})$$

This expression is local since it contains only integration over $\bar{s}$ and $\bar{t}$

Two homotopy operators are related by the field redefinition MV 2016

$$C \to C'(Y|x) = C(Y|x) + \Phi(Y|x)$$

$$\Phi_\eta(Y|x) = \eta\int \frac{dSdT}{(2\pi)^4}\exp iS_A T^A \int d\tau_i \prod_{i=1}^{3} dZ(\tau_i)\delta'\Big(1-\sum_{i=1}^{3}\tau_i\Big) \\ J(\tau_3 s+\tau_1 y, t-\tau_2 y;\bar{y}+\bar{s},\bar{y}+\bar{t})$$

An important consequence of the analysis in the 0-form sector was 2016 that the current coupling constants are proportional to $\eta\bar{\eta}$.

1-form sector

Quadratic corrections in the 1-form sector via standard homotopy

$$D\omega - \frac{i}{4}\Big(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial \overline{y}^{\dot\alpha}\partial \overline{y}^{\dot\beta}}\ \bar C(0,\overline{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0|x)\Big) + \Gamma(w,J) = 0$$

$$\Gamma_{\eta\eta} = \frac{i\eta^2}{8}\int dS\, dT\ \exp(is_\alpha t^\alpha + i\bar s_{\dot\alpha}\bar t^{\dot\alpha})$$

$$\Big(\int_0^1 d\tau\Big[\exp(i(\tau s-t)_\alpha y^\alpha)\Big\{\omega_{L\nu}{}^{\alpha}\omega_L{}^{\beta\nu}(\tau s-t)_\alpha(\tau s-t)_\beta + 2h_\nu{}^{\dot\alpha}\omega_L{}^{\nu\beta}\,(\bar t-\bar s)_{\dot\alpha}(\tau s-t)_\beta\Big\}$$

$$-\Big\{\exp(i(\tau s-t)_\alpha y^\alpha)-1\Big\}\overline{H}^{\dot\alpha\dot\beta}(\bar t-\bar s)_{\dot\alpha}(\bar t-\bar s)_{\dot\beta}\Big]J(\tau s,\, t,\, \bar y+\bar s,\, \bar y+\bar t|x)$$

$$+\int_0^1 d\tau_1\int_0^1 d\tau_2\Big(\exp i(\tau_1 s_\gamma-\tau_2 t_\gamma)y^\gamma\Big)\Big\{\omega_L{}^{\alpha}{}_{\gamma}\omega_L{}^{\gamma\beta}s_\mu t^\mu\tau_1\tau_2\Big(2t_\alpha s_\beta-\tau_1 s_\alpha s_\beta-\tau_2 t_\alpha t_\beta\Big)$$

$$-2\omega_{L\nu}{}^{\beta}h^{\nu\dot\beta}\tau_1\tau_2 s_\mu t^\mu(t_\beta\bar s_{\dot\beta}+s_\beta\bar t_{\dot\beta}) + 2\omega_L^{\alpha\gamma}h^{\mu\nu'}(\tau_1\tau_2-1)(-\tau_1 t_\alpha s_\gamma s_\mu\bar s_{\nu'}+\tau_2 s_\alpha t_\gamma t_\mu\bar t_{\nu'})$$

$$-H^{\alpha\beta}s_\alpha t_\beta(\bar s_{\nu'}\bar t^{\nu'}-2i)(\tau_1\tau_2-1)-\overline{H}^{\dot\alpha\dot\beta}s_\nu t^\nu\Big((\tau_1\tau_2+1)\bar s_{\dot\alpha}\bar t_{\dot\beta}-\tau_1\bar s_{\dot\alpha}\bar s_{\dot\beta}-\tau_2\bar t_{\dot\beta}\bar t_{\dot\alpha}\Big)\Big\}$$

$$J(\tau_1 s,\tau_2 t,\bar y+\bar s,\bar y+\bar t|x)\Big)$$

$$\Gamma = \eta^2\Gamma_{\eta\eta}(w,J)+\bar\eta^2\Gamma_{\bar\eta\bar\eta}(w,J)\ , \qquad \Gamma_{\eta\bar\eta}(w,J)=0$$

Trivialization of η^2-terms

upon the field redefinition

$$\omega(y,\bar{y}|x) \to \omega(y,\bar{y}|x) + \Omega + \Psi\,, \qquad C \to C'(Y|x) = C(Y|x) + \Phi(Y|x)$$

Ω contains Lorentz connection and

$$\Psi = -i\frac{\eta^2}{4}h^{\alpha\dot{\beta}}\int_0^1 d\tau_1\int_0^1 d\tau_2\int dSdT \\ \tau_1 s_\alpha \bar{s}_{\dot{\beta}}\exp(is_\beta t^\beta + i\bar{s}_{\dot{\alpha}}\bar{t}^{\dot{\alpha}} + i(\tau_1 s_\gamma - \tau_2 t_\gamma)y^\gamma)J(\tau_1 s, \tau_2 t, \bar{y}+\bar{s}, \bar{y}+\bar{t})$$

Field equations in the 1-form sector take the form

$$\mathcal{D}_{ad}\omega = \frac{i}{4}\Big(\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\beta}}}\;C(0,\bar{y}|x) + \bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha\partial y^\beta}\;C(y,0|x)\Big) + \Gamma'_{\eta\bar{\eta}}$$

$$\Gamma'_{\eta\bar{\eta}} = \frac{i}{8}\eta\bar{\eta}\int dSdT\exp iS_A T^A\int d^3\bar{\tau}d^3\tau\,\Big\{\prod_{i=1}^3\theta(\bar{\tau}_i)\theta(\tau_3)\delta(x)\delta'(\bar{x})\delta(\tau_1)\delta(\tau_2)\overline{H}^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{\dot{\alpha}}\bar{\partial}_{\dot{\beta}} \\ + \prod_{i=1}^3\theta(\tau_i)\theta(\bar{\tau}_3)\delta'(x)\delta(\bar{x})\delta(\bar{\tau}_1)\delta(\bar{\tau}_2)H^{\alpha\beta}\partial_\alpha\partial_\beta\Big\}\,J(\tau_3 s+\tau_1 y, t-\tau_2 y; \bar{\tau}_3\bar{s}+\bar{\tau}_1\bar{y}, \bar{t}-\bar{\tau}_2\bar{y})$$

$$x = 1 - \sum_{i=1}^3 \tau_i\,, \qquad \bar{x} = 1 - \sum_{i=1}^3 \bar{\tau}_i$$

From nonlocal to local deformation

$$\mathcal{D}_{ad}X = \Gamma'_{\eta\bar\eta} - \Gamma^{loc}_{\eta\bar\eta}$$

where

$$\Gamma^{loc}_{\eta\bar\eta} = \frac{i}{8}\eta\bar\eta \bar H^{\dot\alpha\dot\beta}\bar\partial_{\dot\alpha}\bar\partial_{\dot\beta}\int d^2\bar\tau d^2\tau \delta(1-\bar\tau_3-\bar\tau_4)\delta'(1-\tau_1-\tau_2)\theta(\tau_1)\,\theta(\tau_2)\,\theta(\bar\tau_3)\,\theta(\bar\tau_4)$$
$$\frac{1}{(\bar\tau_4)^2}\exp i\Big(\bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\Big)J(\tau_1 y,-\tau_2 y;\bar\tau_4\tau_2\bar y,-\bar\tau_4\tau_1\bar y)$$
$$+\frac{i}{8}\eta\bar\eta H^{\alpha\beta}\partial_\alpha\partial_\beta\int d^4\tau\delta(1-\tau_3-\tau_4)\delta'(1-\tau_1-\tau_2)\theta(\tau_1)\,\theta(\tau_2)\,\theta(\tau_3)\,\theta(\tau_4)$$
$$\frac{1}{(\tau_4)^2}\exp i\Big(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha}\Big)J(\tau_4\tau_1 y,-\tau_4\tau_2 y;\tau_2\bar y,\tau_1\bar y)$$

$$X(J) = \frac{i}{8}\eta\bar\eta\int d\bar\tau_3 d\bar\tau_4 d^4\tau\delta(1-\tau_3-\tau_4)\delta(1-\bar\tau_3-\bar\tau_4)\delta'(1-\tau_1-\tau_2)\,h^{\alpha\dot\beta}\partial_\alpha\bar\partial_{\dot\beta}$$
$$\frac{(1-\tau_3\bar\tau_3)}{\tau_4\bar\tau_4}\exp i\Big(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha}+\bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\Big)J(\tau_4\tau_1 y,-\tau_4\tau_2 y;\bar\tau_4\tau_2\bar y,-\bar\tau_4\tau_1\bar y)\,\Upsilon,$$

Application of the alternative HS homotopy operator ∂_{HS} gives

$$\Gamma_{\eta\eta} = \Gamma_{\bar\eta\bar\eta} = 0\,, \qquad \Gamma^{loc}_{\eta\bar\eta} \neq 0$$

Canonical current interactions

Being local $\Gamma^{loc}_{\eta\bar{\eta}}$ contains higher derivatives (improvements) and nonzero torsion-like terms. The last step is to bring local current interactions by a local field redefinition to the canonical form with the minimal number of derivatives and zero torsion. The final result for the vertex is

$$\tilde{\Gamma}^{loc}_{\eta\bar{\eta}} = \frac{i}{8}\eta\bar{\eta}\int \frac{d^3\rho d^2\tau}{\rho_2^2}\theta(\rho_1)\theta(\rho_2)\theta(\rho_3)\delta'(1-\rho_1-\rho_2-\rho_3)\theta(\tau_1)\theta(\tau_2)\delta'(1-\tau_1-\tau_2)$$
$$\int_{|w|=1}\frac{dw}{2\pi i w}\Big\{\int d\bar{u}d\bar{v}\bar{H}^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{\dot{\alpha}}\bar{\partial}_{\dot{\beta}}\Big[(1+w^{-1})\exp\left(i\rho_1\rho_3(\rho_1+w^2\rho_3)^{-1}\bar{u}_{\dot{\gamma}}\bar{v}^{\dot{\gamma}}\right)$$
$$J(w^{-1}\tau_2 y, -w\tau_1 y, w\rho_2\tau_1\bar{y}+\bar{u}, -w\rho_2\tau_2\bar{y}-\bar{v}|x)$$
$$+(1+w)\int dsdt\exp\left(i(\rho_1-\rho_3 w^2)\bar{u}_{\dot{\gamma}}\bar{v}^{\dot{\gamma}}+i\rho_3 s_\gamma t^\gamma - i\rho_3 w^2\rho_2[\tau_2\bar{u}_{\dot{\gamma}}-\tau_1\bar{v}_{\dot{\gamma}}]\bar{y}^{\dot{\gamma}}\right)$$
$$J(w^{-1}\tau_2 y+s, -w^{-1}\tau_1 y - t, w(\rho_2\tau_1\bar{y}+\bar{u}), -w(\rho_2\tau_2\bar{y}+\bar{v})|x)$$
$$-\exp\left(i\rho_1\bar{u}_{\dot{\gamma}}\bar{v}^{\dot{\gamma}}\right)J(w^{-1}\tau_2 y, -w^{-1}\tau_1 y, w(\rho_2\tau_1\bar{y}+\bar{u}), -(w\rho_2\tau_2\bar{y}+\bar{v})|x)\Big]+c.c.\Big\}$$

Boundary limit

$$C^{j\,1-j}(y,\bar y|\mathbf{x},\mathbf{z}) = \mathbf{z}\,\mathsf{exp}(y_\alpha \bar y^\alpha) T^{j\,1-j}(w,\bar w|\mathbf{x},\mathbf{z})\,, \qquad w^\alpha = \mathbf{z}^{1/2} y^\alpha \quad \bar w^\alpha = \mathbf{z}^{1/2}\bar y^\alpha$$

where $T^{j\,1-j}$ are associated with the boundary currents.

The contribution of HS connections at the boundary cannot be neglected except for the boundary conditions MV 2012

$$\bar\eta T_+^{j\,1-j}(y,\bar y|\mathbf{x},0) - \eta T_-^{1-j\,j}(-i\bar y, iy|\mathbf{x},0) + \dots = 0\,,$$

where T_+ and T_- are the positive and negative helicity parts of $T(y,\bar y|x)$.

This is the condition that boundary conformal HS gauge fields are not excited, most relevant to the subject of this conference

For some reason this language is holographically invariantly non-convincing for the majorities of most of the bulk and boundary communities

Hence I will use today a different approach which is equivalent at least in the lowest order responsible for current interactions and cubic correlators.

In the local frame, the HS equations have the form

$$D\omega - \frac{i}{4}\Big(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\ \bar{C}(0,\overline{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\ C(y,0|x)\Big) + \widetilde{\Gamma}_{\eta\bar\eta}(J)^{\,loc} = 0$$

Though the nonlinear terms do not depend on the phase φ, the linear ones do. To go to the standard frame it is convenient to make the redefinition

$$\bar{C}'_- = \eta C_-\,, \qquad C'_+ = \bar\eta C_+$$

where C_+ and $\bar{C}'_-$ are the self-dual and anti-self-dual parts of the 0-forms

Removing the phase φ from the linear terms this duality transformation brings the fields C and hence the current interaction term to the phase-dependent form. In these variables, the boundary conditions and, hence the whole holographic prescription is standard while the interaction terms acquire the Maldacena-Zhiboedov-Giombi-Yin form

$$V = cos^2(\varphi)V_b + sin^2(\varphi)V_f + \frac{1}{2}sin(2\varphi)V_o$$

Conclusions

Quadratic corrections to nonlinear HS equations handled with appropriate HS homotopy operators are local, containing a finite number of derivatives for any three spins in the vertex.

Remarkably, current interactions in HS theory are proportional to $\eta\bar{\eta}$

The sign in front of the stress tensor is always positive

Confirms that the theory with $\bar{\eta} = 0$ is self-dual: No current interactions can be expected in the self-dual sector of the HS theory
= triviality of the three-particle amplitude with positive helicities

The seemingly paradoxical issue with HS holography is resolved via the φ-dependence of linear terms

To do: work in progress

Compute directly the boundary three-point function

Extend the analysis of locality and HS homotopy operator to all orders of HS theory