Dualities in $2 + 1$ Dimensions and their Global Symmetries

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Dualities in $(2+1)d$

(This will be a continuation of Seiberg's talk)

IR dualities between Chern-Simons gauge theories with matter in fundamental rep

[Aharony 15; Hsin, Seiberg 16; Aharony, FB, Hsin, Seiberg 16]

scalars ϕ with $\lvert\phi\rvert^4$ interactions		fermions ψ	
$SU(N)_k$ with N_f ϕ	$\longleftrightarrow$	$U(k)_{-N+\frac{N_f}{2}}$	with N_f ψ
$U(N)_k$ with N_f ϕ	$\longleftrightarrow$	$SU(k)_{-N+\frac{N_f}{2}}$	with N_f ψ
$U(N)_{k,k\pm N}$ with N_f ϕ	$\longleftrightarrow$	$U(k)_{-N+\frac{N_f}{2},-N\mp k+\frac{N_f}{2}}$	with N_f ψ
$USp(2N)_k$ with N_f ϕ	$\longleftrightarrow$	$USp(2k)_{-N+\frac{N_f}{2}}$	with N_f ψ
$SO(N)_k$ with N_f $\phi_{\mathbb{R}}$	$\longleftrightarrow$	$SO(k)_{-N+\frac{N_f}{2}}$	with N_f $\psi_{\mathbb{R}}$

valid for N_f less than a bound.

Motivated by very different ideas in condensed matter, SUSY QFT and AdS/CFT

[Aharony, Barkeshli, Giombi, Gur-Ari, Jain, Karch, Maldacena, McGreevy, Metlitski, Minwalla, Murugan, Nastase, Prakash, Radicevic, Seiberg, Senthil, Sharma, Son, Takimi, Tong, Trivedi, Vishwanath, Wadia, Wang, Witten, Xu, Yacoby, Yin, Yokoyama, Zhiboedov, ...]

Dualities in $(2+1)d$

Very interesting consequences, *e.g.*:

- Bosonization of free fermions: (and similarly for Wilson-Fisher bosons)

$$U(N)_1 \text{ with } \phi \quad \longleftrightarrow \quad \text{free Dirac } \psi$$
$$SO(N\geq 3)_1 \text{ with } \phi_{\mathbb{R}} \quad \longleftrightarrow \quad \text{free Majorana } \psi_{\mathbb{R}}$$

- IR quantum enhanced symmetries:

$$U(1)_2 \text{ with } \phi \;\leftrightarrow\; U(1)_{-\frac{3}{2}} \text{ with } \psi \;\leftrightarrow\; SU(2)_1 \text{ with } \phi \;\leftrightarrow\; SU(2)_{-\frac{1}{2}} \text{ with } \psi$$

  have $SO(3)$ global symmetry in the IR

- IR quantum time-reversal symmetry:

$$U(1)_{\frac{1}{2}} \text{ with } \psi \quad \longleftrightarrow \quad U(1)_{-\frac{1}{2}} \text{ with } \psi$$

  The duality is time reversal

Tests

We cannot prove these dualities. . .   but we can test them.

- Consistency under mass deformations
- large N, k (with N/k fixed) matches AdS/CFT results

  [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 11; Aharony, Gur-Ari, Yacoby 12; . . .]
- small $N = k = N_f = 1$ matches particle/vortex duality and similar ones

  [Peskin 78; Dasgupta, Haperin 81; Barkeshli, McGreevy 14; Son 15; Potter, Serbyn, Vishwanath 15; Wang, Senthil 16; Karch, Tong 16; Seiberg, Senthil, Wang, Witten 16]
- $N_f = 0$ matches level/rank dualites

  [Aharony 15; Hsin, Seiberg 16; FB, Hsin, Seiberg 16]
- Sometimes related to SUSY dualities by mass deformations

  [Jain, Minwalla, Yokoyama 13; Gur-Ari, Jacoby 15; Kachru, Mulligan, Torroba, Wang 16]

SUSY dualities in $(2+1)d$

Compare with IR dualities among SUSY gauge theories

- *E.g.*: $\mathcal{N} = 2$ SUSY (for $k < N_f/2$) [FB, Closset, Cremonesi 11]

$$U(N)_k \text{ with } N_f \text{ fund.} \quad \longleftrightarrow \quad U(N_f - N)_{-k} \text{ with } N_f \text{ antifund.}$$

This is just a tiny piece in a much bigger story:

- SUSY dualities range from $(1+1)d$ to $(5+1)d$
- There are *myriads of examples* with various amounts of SUSY
  Such a huge intricate net is self-consistent under all sorts of deformations
- Map many operators across the dualities
- Many "protected" quantities can be computed exactly at strong coupling:
  - partition functions
  - VEVs of local and extended defect operators
  - operator countings and anomalous dimensions

  They match across the dualities (due to sophisticated mathematical identities)

What other tests without SUSY?

So far, nothing similar without SUSY

- It is important to find more sophisticated tests of the dualites.

$\rightsquigarrow$ Aspects related to the global symmetry and anomalies

Anomalies are among the few “protected” quantities in the non-SUSY world

Faithful global symmetry

1. Identify the global symmetry

Important to identify the symmetry that acts *faithfully* on gauge-invariant oper's

Dual theories should have the same *faithful* global symmetry

$\rightarrow$ not only match summetry *currents* (algebra) but also symmetry *group*

- *E.g.*: $SU(N)_{k-\frac{N_f}{2}}$ with N_f ψ

Naively: $U(N_f)$ acting on matter fields (besides $\mathbb{Z}_2^{\mathcal{C}}$)

However $\mathbb{Z}_N \subset U(N_f)$ is already gauged

$\Rightarrow$ gauge-invariant operators only give reps of $U(N_f)/\mathbb{Z}_N$

Faithfully-acting symmetry is $U(N_f)/\mathbb{Z}_N$

Faithful global symmetry

Things get more interesting when there are magnetic symmetries:

(with CS int.) Monopoles $\mathcal{M}$ have gauge charges and need to be dressed

- *E.g.*: $U(k)_{-N}$ with N_f ϕ

Naively: $SU(N_f) \times U(1)_{\text{mag}}$ (besides $\mathbb{Z}_2^{\mathcal{C}}$)

However $\mathcal{M}$ has gauge charge $-N$

$\Rightarrow$ $\mathcal{M}\phi^N$ has $SU(N_f)$ N_f-ality N

(correlation between magnetic charge and flavor rep)

$$\mathbb{Z}_{N_f}\!: (e^{-\frac{2\pi i}{N_f}}, e^{\frac{2\pi i N}{N_f}}) \;\leftarrow\; \frac{SU(N_f) \times U(1)_{\text{mag}}}{\mathbb{Z}_{N_f}} = U(N_f)/\mathbb{Z}_N$$

- The two examples are dual, and the *faithful global symmetry* matches

Background fields

2. A theory with global symmetry G can be coupled to background gauge fields

This is a very powerful tool in QFT

$\to$ it provides observables (*e.g.* all sorts of partition functions)

- Suppose $G_{\text{faithful}} = G_{\text{naive}}/Z_{\text{discrete}}$

  G_{faithful} bundles are more general than G_{naive} bundles

If we misidentify the symmetry, we miss important observables!

"Twisted" observables

- *E.g.*: $SU(2)_k$ with N_f ϕ

Naive global symmetry: $USp(2N_f) \equiv Sp(N_f)$

The matter really couples to $$\frac{SU(2) \times USp(2N_f)}{\mathbb{Z}_2} \qquad \Rightarrow \; G = \frac{USp(2N_f)}{\mathbb{Z}_2}$$

Think of it as two *correlated* bundles:

- Background is $USp(2N_f)$ bundle $\Rightarrow$ dynamical $SU(2)$ bundles
- Background is $\frac{USp(2N_f)}{\mathbb{Z}_2}$ bundle $\Rightarrow$ dynamical $\frac{SU(2)}{\mathbb{Z}_2} = SO(3)$ bundles

Correlation: $$w_2\Big(USp(2N_f)/\mathbb{Z}_2\Big) = w_2\Big(SO(3)\Big)$$

(second Stiefel-Whitney classes of the bundles)

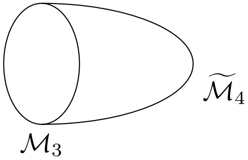
- Activating the background, we probe nontrivial *twisted sectors* of the theory

't Hooft anomalies

3. Global symmetry G can have 't Hooft anomalies:

- correlation functions at separated points are G-invariant, but the contact terms *cannot* be
- the system *cannot* be consistently coupled to G background fields, or is *not* invariant under G gauge transformations

Often, the lack of gauge invariance can be eliminated by coupling to a higher-dimensional bulk theory:



't Hooft anomalies

To detect 't Hooft anomalies, we try to couple to background G bundles

$\rightarrow$ introduce Chern-Simons counterterms L for background gauge fields

Such counterterms L should be properly quantized

- Quantization depends on G and is affected by quotients
- Quantization depends on dynamical CS terms k

It might happen that there is *no* consistent value of L !

$\longrightarrow$ 't Hooft anomaly for G

- Remark: this anomaly is particularly relevant in $(2+1)d$, since conserved currents are not anomalous

't Hooft anomalies

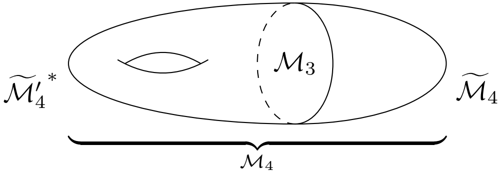
Two options:

- only consider a subgroup or a multiple cover of G
- extend G gauge fields to the bulk (dynamical fields are not extended!)

Quantify the anomaly by the dependence of the partition function on the bulk extension (SPT phases in CM)

$\rightarrow$ characteristic classes of G bundles on closed $\mathcal{M}_4$
(*e.g.* $F \wedge F$ or discrete θ-terms)

$$S_{\text{anom}} = \int_{\mathcal{M}_4} G\text{-class}$$



- Dual theories should have the same dependence $\Rightarrow$ 't Hooft anomaly matching

Example I

E.g.: $SU(2)_k$ with N_f ϕ

Faithful symmetry $G = USp(2N_f)/\mathbb{Z}_2$ (for $N_f = 1$: $G = \frac{SU(2)}{\mathbb{Z}_2} = SO(3)$)

We couple to background G bundles, and choose a CS counterterm L:

$$\frac{SU(2)_k \times USp(2N_f)_L}{\mathbb{Z}_2} \quad \text{with } \phi$$

Quantization conditions on L: $L \in \mathbb{Z}\,, \qquad k + N_f L \in 2\mathbb{Z}$

- For k odd, N_f even, there are *no solutions*: anomaly!

$$S_{\text{anom}} = \pi \int_{\mathcal{M}_4} \frac{w_2\big(USp(2N_f)/\mathbb{Z}_2\big)^2}{2} \qquad \text{(Pontryagin square)}$$

Dependence on bulk extension of G bundles is mild: $e^{iS_{\text{anom}}} = \pm 1$

Example II

Perform 't Hooft anomaly matching on the proposed dualities

- *E.g.*: $USp(2N)_k$ with N_f ϕ $\longleftrightarrow$ $USp(2k)_{-N+\frac{N_f}{2}}$ with N_f ψ

$(N_f \leq N)$

Faithful symmetry $G = USp(2N_f)/\mathbb{Z}_2$

The CS counterterm L is matched across the duality:

$$\frac{USp(2N)_k \times USp(2N_f)_L}{\mathbb{Z}_2} \text{ with } \phi \quad \leftrightarrow \quad \frac{USp(2k)_{-N+\frac{N_f}{2}} \times USp(2N_f)_{L+\frac{k}{2}}}{\mathbb{Z}_2} \text{ with } \psi$$

- Same quantization condition: $Nk + N_f L \in 2\mathbb{Z}$
- For Nk odd, N_f even, same anomaly: $S_{\text{anom}} = \pi \int_{\mathcal{M}_4} \frac{w_2^2}{2}$

Example III

QED with two fermions: $U(1)_0$ with $N_f = 2$ ψ

- Manifest symmetry: $\frac{SU(2)^X \times U(1)^Y_{\text{mag}} \rtimes \mathbb{Z}_2^{\mathcal{C}}}{\mathbb{Z}_2} = \frac{SU(2)^X \times O(2)^Y}{\mathbb{Z}_2}$

  Besides, $\mathcal{T}$-reversal invariant with anomaly:

$$\mathcal{T}\Big[\mathcal{L}_0(X,Y)\Big] = \mathcal{L}_0(X,Y) + SU(2)^X_1 + U(1)^Y_2$$

- Theory enjoys self-duality where $U(1)^Y_{\text{mag}} \leftrightarrow U(1)^X \subset SU(2)^X$

  [Xu, You 15; Karch, Tong 16; Hsin, Seiberg 16]

  $\Rightarrow$ IR fixed point has larger symmetry: $\frac{SU(2)^X \times SU(2)^Y}{\mathbb{Z}_2} \rtimes \mathbb{Z}_2^{\mathcal{D}} \times \mathbb{Z}_2^{\mathcal{T}} = O(4) \times \mathbb{Z}_2^{\mathcal{T}}$

  Example where neither description has the full symmetry

Example III

CS counterterms necessary for self-duality are not properly quantized in $O(4)$:

- Couple only to $Pin^{\pm}(4)$ backgrounds (no $\mathbb{Z}_2$ quotient)
- Shift CS terms and couple only to $SO(4)$ backgrounds (no $\mathbb{Z}_2^{\mathcal{D}}$ self-duality)
- Couple to a bulk (no pure $(2+1)d$ theory)

- Recover $O(4)$ through bulk:

Move $S_B = SU(2)^X_{-1} = -\frac{1}{4\pi}\int \mathrm{Tr}\, F^X_{SU(2)} \wedge F^X_{SU(2)}$ to the bulk

Full system has $O(4)$ invariance, but there is a mild dependence on bulk extension of $O(4)$ bundle

$\rightarrow$ 't Hooft anomaly: $e^{iS_B} = \exp\left(i\pi \int_{\mathcal{M}_4} \frac{w_2\big(SO(4)\big)^2}{2}\right) = \pm 1$

- Recover $O(4) \times \mathbb{Z}_2^{\mathcal{T}}$:

Add $S'_B = \frac{1}{8\pi}\int \mathrm{Tr}\left(F^X_{SU(2)} \wedge F^X_{SU(2)} + F^Y_{SU(2)} \wedge F^Y_{SU(2)}\right)$

Generating new dualities

4. If $G' \subset G$ has no 't Hooft anomaly, we can make it dynamical

Possibly, couple the new dynamical fields to new background fields

- *E.g.*: Witten's S, T operations on theories with a $U(1)$ global symmetry

[Witten 03]

$\rightarrow$ $SL(2,\mathbb{Z})$ action on a space of theories

Performing this operation on a pair of dual theories

$\rightsquigarrow$ *new dual pair* (and so on...)

$\Rightarrow$ Duality-generating technique (Deserves further study!)

It is important that dynamical fields do not depend on bulk $\Leftrightarrow$ no anomaly

Conclusions

- The faithful global symmetry and its coupling to background fields is a powerful diagnostics of a QFT

  $\rightarrow$ Constraints/tests on conjectured dualities

- Anomalies are some of the few examples of "protected quantities" in non-SUSY QFTs

- Understanding anomalies is crucial if we want to gauge a symmetry to obtain new theories/dualities