Duality in 2 + 1 Dimensions

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IAS

NS and E. Witten, arXiv:1602.04251;

NS, T. Senthil, C. Wang, and E. Witten, arXiv:1606.01989;

P.-S. Hsin, NS, arXiv:1607.07457

O. Aharony, F. Benini, P.-S. Hsin, NS, arXiv:1611.07874

F. Benini, P.-S. Hsin, NS, arXiv:1702.07035

Duality in continuum QFT

In this talk I will limit myself to continuum QFT.

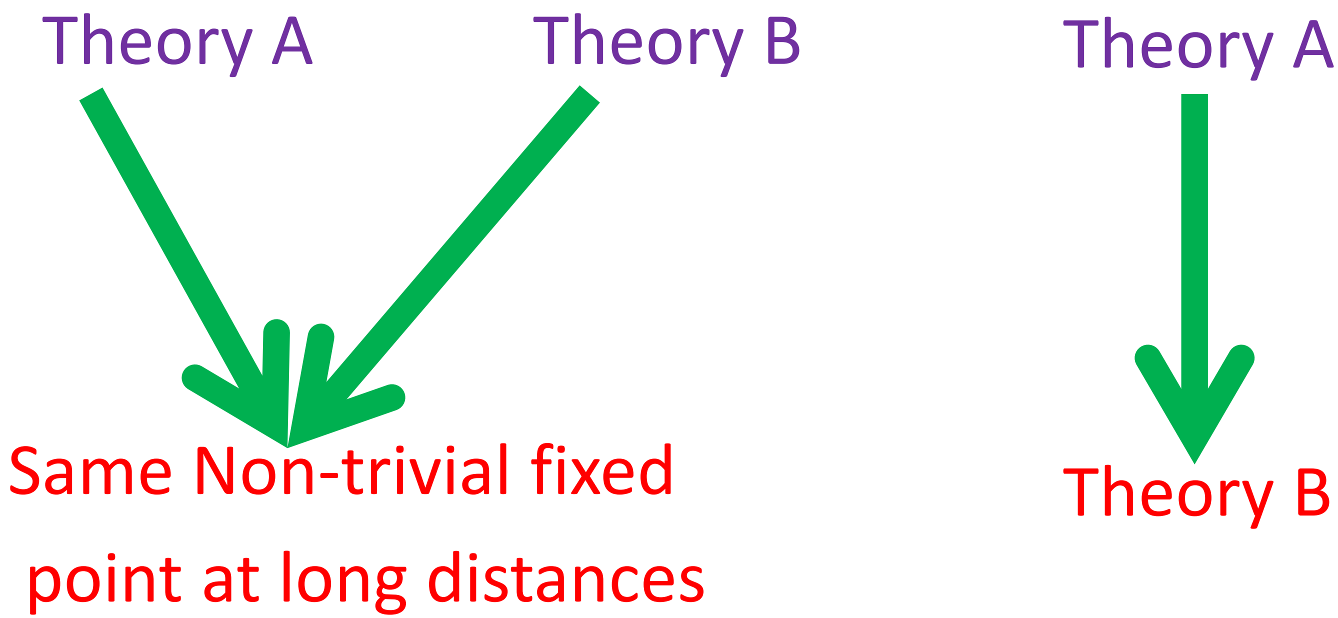
This is a renormalizable continuum QFT perturbed by a small number of relevant operators or infinitesimal irrelevant operators – no lattice.

We will discuss duality only in this sense.

Duality in continuum QFT

Different kinds of duality including

- Exact duality as in the 2d Ising model or $\mathcal{N} = 4$ SUSY
- IR duality: two different theories with the same IR behavior (e.g. particle/vortex in 3d, $\mathcal{N} = 1$ SUSY)



Three (almost) independent lines of development
The unity of physics

- The condensed matter, $3d$ quantum field theory route
- The supersymmetric route
- The AdS/CFT, large N route

They recently converge on the same set of ideas.

We will not review it here.

Important preliminary comments

- All gauge groups are compact in the HE sense. This does not mean that there is a monopole operator in the Lagrangian.
- All Chern-Simons terms have to be properly quantized.
- A fermion kinetic term $i\bar{\Psi}\not{D}_A\Psi$

  is what is often written (somewhat carelessly) as

$$i\bar{\Psi}\not{D}_A\Psi - \frac{1}{8\pi}AdA$$

We will refer to such a fermion as coupled to $U(1)_{-\frac{1}{2}}$.

Conjectured boson/fermion dualities

Many references. These are some of the recent ones.

…; Giombi, Yin; Aharony, Gur-Ari, Yacoby; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin; Maldacena, Zhiboedov; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama; Minwalla, Yokoyama; Yokoyama; Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama; Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama; Jain, Minwalla, Yokoyama; Gur-Ari, Yacoby; Son; Wang, Senthil; Metlitski, Vishwanath; Barkeshli, McGreevy; Radicevic; Aharony; Karch, Tong; NS, Senthil, Wang, Witten; Hsin, NS; Kachru, Mulligan, Torroba, Wang; Metlitski, Vishwanath, Xu; Aharony, Benini, Hsin, NS; Benini, Hsin, NS …

Conjectured boson/fermion dualities

N_f scalars at $\|\Phi\|^4$ point coupled to		N_f fermions coupled to
• $SU(N)_k$	$\leftrightarrow$	$U(k)_{-N+N_f/2,-N+N_f/2}$
• $U(N)_{k,k}$	$\leftrightarrow$	$SU(k)_{-N+N_f/2}$
• $U(N)_{k,k+N}$	$\leftrightarrow$	$U(k)_{-N+N_f/2,-N-k+N_f/2}$
• $U(N)_{k,k-N}$	$\leftrightarrow$	$U(k)_{-N+N_f/2,-N+k+N_f/2}$
• $SO(N)_k$	$\leftrightarrow$	$SO(k)_{-N+N_f/2}$
• $Sp(N)_k$	$\leftrightarrow$	$Sp(k)_{-N+N_f/2}$

The scalars are in a generalized Wilson-Fisher fixed point or a gauged version of it.

N_f cannot be too large (different bounds in the different cases).

Conjectured boson/fermion dualities

- Supported and partly motivated by
  - AdS/CFT and large N computations ($N, k \to \infty$ with finite N/k)
  - similar supersymmetric dualities
  - particle/vortex duality and suggested boson/fermion dualities in Condensed Matter
- Can assume some of them and derive others
- Consistent with mass deformations
- Related to level/rank duality (set $N_f = 0$)
- Global symmetry, ‘t Hooft anomaly matching

Some special cases (free complex fermion)

Substitute $N_f = k = 1$ in

$$U(N)_{k,k} \quad with \;\; \Phi \qquad \leftrightarrow \qquad SU(k)_{-N+N_f/2}\, with \;\; \Psi$$

N complex scalars with $|\Phi|^4$ coupled to $U(N)_{1,1}$

↓

a free complex fermion Ψ

Bosonization or Fermioniziation

For $N = 1$ the UV theory is the gauged $O(2)$ WF theory with a CS term

Some special cases (free complex fermion)

N complex scalars with $|\Phi|^4$ coupled to $U(N)_{1,1}$

↓

a free complex fermion Ψ

Duality is most interesting when one of the sides is free

$\Psi \quad \leftrightarrow \quad \mathcal{M}_b$ is a monopole operator of $\mathrm{U}(N)_{1,1}$

Mass deformation ($\bar{\Psi}\Psi \quad \leftrightarrow \quad |\Phi|^2$) depends on the sign

Time reversal symmetry $\mathcal{T}$ with anomaly.

$\mathcal{T}$ not present in the UV. Nonlocal in the bosonic variables

Some special cases (free real fermion)

[Metlitski, Vishwanath, Xu; Aharony, Benini, Hsin, NS]

Substitute $N_f = k = 1$ in

$$SO(N)_k \; with \;\; \phi \quad \leftrightarrow \quad SO(k)_{-N+N_f/2} \; with \;\; \psi$$

N real scalars with ϕ^4 coupled to SO$(N)_1$

a free real (Majorana) fermion ψ

Bosonization/fermioniziation for Majorana fermions

Some special cases (free real fermion)

N real scalars with ϕ^4 coupled to SO$(N)_1$

a free real (Majorana) fermion ψ

$\psi \quad \leftrightarrow \quad \mathcal{M}_b$ is a $\boldsymbol{Z}_2$ monopole operator of $SO(N)_1$

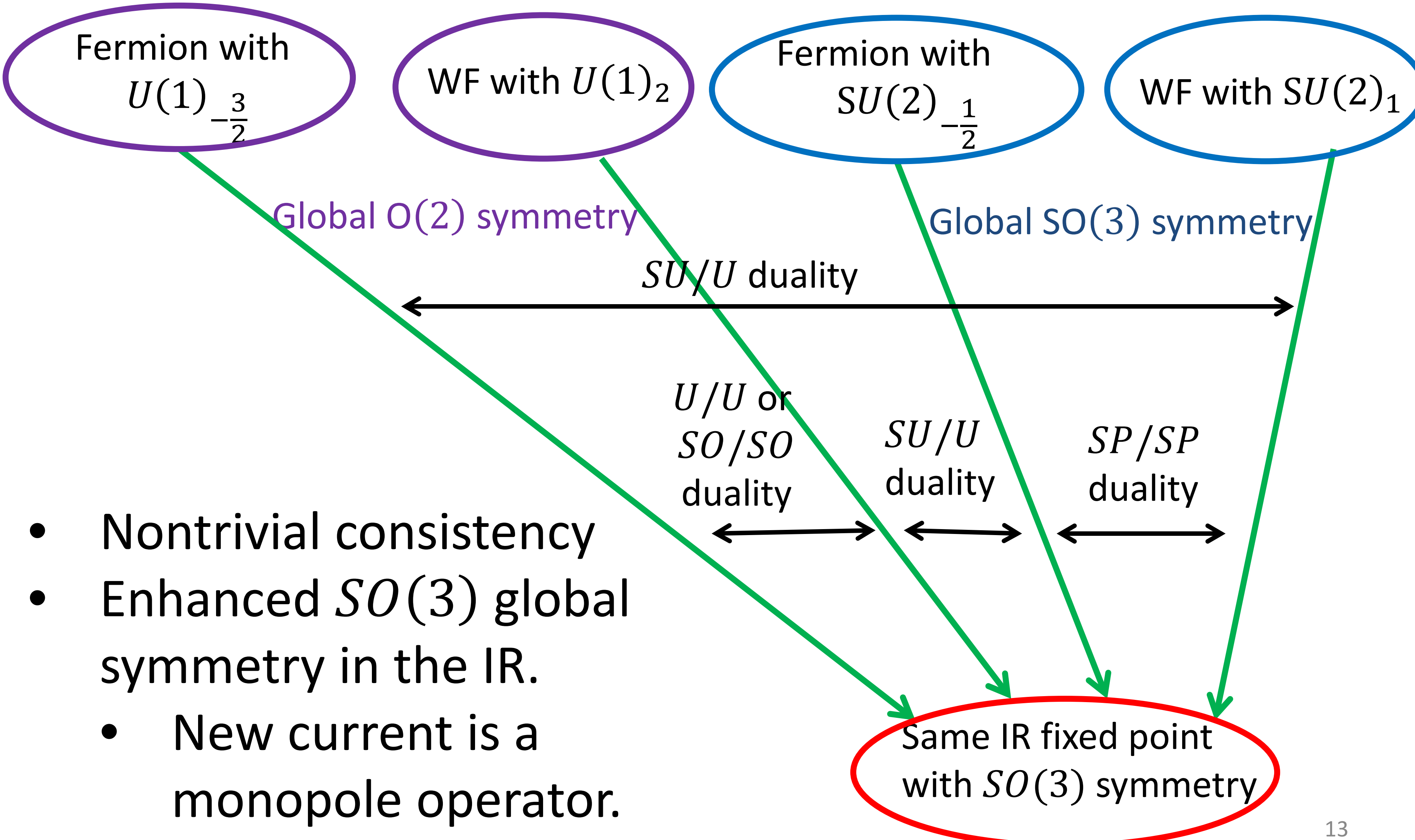
Charge conjugation $\leftrightarrow$ monopole number modulo 2

Mass deformation ($\psi\psi \quad \leftrightarrow \quad \phi^2$) depends on the sign

$\mathcal{T}$ with gravitational anomaly

$\mathcal{T}$ not present in the UV. Nonlocal in the bosonic variables

Example of a web of dualities



- Nontrivial consistency
- Enhanced $SO(3)$ global symmetry in the IR.
  - New current is a monopole operator.

Review of special cases

$U(1)_0$ with Φ $\leftrightarrow$ $\widehat{\Phi}$ XY/WF standard particle/vortex duality

$\leftrightarrow$ $U(1)_{\frac{1}{2}}$ with Ψ QED

$U(1)_1$ with Φ $\leftrightarrow$ Ψ free bosonization/fermionization

$U(1)_2$ with Φ $\leftrightarrow$ … quantum $SO(3)$ global symmetry

In all these examples the scalars have a $|\Phi|^4$ interaction and we tune to a nontrivial fixed point

The monopole operator of $U(1)_k$ has spin $\frac{k}{2}$. It is $\widehat{\Phi}$, a free fermion Ψ, and a conserved current in these cases.

Couple to classical background

An important tool – couple to classical background fields.

We can always couple to a metric.

If there is a global symmetries, e.g. $U(1)_A$, couple to classical background gauge fields A.

For the duality to be valid we might need to add to one side of the duality a gravitational Chern-Simons counterterm or a Chern-Simons counterterm for the classical gauge fields, e.g. $\frac{K}{4\pi}AdA$ with integer K.

More dualities

Given a duality $\mathcal{L}_1 \leftrightarrow \mathcal{L}_2$

we can find new dualities by

- adding the same CS counter term in classical fields to the two sides
- promoting the classical fields to quantum fields and perhaps add new background fields
- Repeat

For $U(1)$ these are Witten's T and S operations on *2+1d* theories.

More dualities

This way we can assume any one of

N_f scalars at $\|\Phi\|^4$ point coupled to		N_f fermions coupled to
• $SU(N)_k$	$\leftrightarrow$	$U(k)_{-N+N_f/2,-N+N_f/2}$
• $U(N)_{k,k}$	$\leftrightarrow$	$SU(k)_{-N+N_f/2}$
• $U(N)_{k,k+N}$	$\leftrightarrow$	$U(k)_{-N+N_f/2,-N-k+N_f/2}$
• $U(N)_{k,k-N}$	$\leftrightarrow$	$U(k)_{-N+N_f/2,-N+k+N_f/2}$

and derive the other three.

More dualities

In many cases (as above) more than one description:

Bosonic ↔ Fermionic ↔ Bosonic’

or

Fermionic ↔ Bosonic ↔ Fermionic’

So assuming these dualities leads to boson/boson or fermion/fermion dualities

e.g. …

More dualities

As a check, derive the standard particle/vortex duality

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad |D_b\hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi}Bdb$$

Derive a fermion/fermion duality (related to [Son; Wang, Senthil; Metlitski, Vishwanath])

$$i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow$$

$$i\bar{\chi}\not{D}_a\chi - \frac{2}{4\pi}bdb + \frac{1}{2\pi}adb + \frac{1}{2\pi}Adb - \frac{1}{4\pi}AdA$$

Note: the dynamics in the RHS is not that of QED (different critical exponents).

Global symmetries
(more details in Benini's talk)

Identify the global properties of the global symmetry.

For example, $SU(2)$ with N_f scalars Φ has a global $Sp(N_f)$ symmetry.

More precisely, Φ couple to $\left(SU(2) \times Sp(N_f)\right)/\boldsymbol{Z}_2$.

Hence, the global symmetry that acts faithfully on gauge invariant operators is

$$Sp(N_f)/\boldsymbol{Z}_2$$

e.g. for $N_f = 1$ it is $SO(3)$ rather than $SU(2)$.

Global symmetries
(more details in Benini's talk)

Couple the global symmetry to classical background fields.

A quotient in the global symmetry, e.g. $Sp(N_f)/\mathbf{Z}_2$ allows us to explore the system in more subtle backgrounds that are not $Sp(N_f)$ bundles.

Typically, there are anomalies in these more subtle backgrounds.

Nontrivial ‘t Hooft anomaly matching conditions on the duality.

Summary

- Duality is ubiquitous in physics.
- Ideas in different branches of physics are useful in other branches. The unity of physics.
- A rich duality web in $2+1d$ physics. Assuming some of these dualities leads to others. Many cross-checks.
  - Boson/boson
  - Boson/fermion
  - Fermion/fermion
- Many more dualities are likely to be present
  - Use the same techniques to find others
  - Find new dualities that do not follow trivially from these