Dualities in 2+1dimensions

Andreas Karch (University of Washington, Seattle)

PCTS workshop on “New Developments in Conformal Field Theory Above Two Dimensions”

Based on work with David Tong (Cambridge) and Brandon Robinson

(related work by Seiberg, Senthil, Witten and Wang)

(some overlap with Murugan, Nastase)

March 8, 2017

Combined: CFT & QHE Session

Wednesday, March 8, 2017

8:30-8:55am	**Continental Breakfast/welcome**
9:00-9:45	"Duality in 2+1d" **Nathan Seiberg, IAS**
9:45-10:30	"Fixed-point collisions and tensorial order parameters in some relativistic and non-relativistic field theories" **Igor Herbut, Simon Fraser University**
10:30-11:00	**Coffee Break**
11:00-11:45	"Dualities in 2+1 dimensions" **Andreas Karch, University of Washington**
11:45-12:30	"talk title" **Speaker TBA**
12:30-2:00	**Lunch at PCTS, Jadwin Hall, Fourth Floor**

Outline

- **Brief Recap of Basic Picture (Our Version)**

(w/ David Tong)

- **Novel Generalizations (large Nf QED and its quiver dual)**

(w/ David Tong and Brandon Robinson)

- **Preliminary Results from ongoing work**

(w/ David Tong and Carl Turner)
(w/ Kristan Jensen)

Low energy description of QHE from “Flux Attachment”

Fermion + flux

Boson

…and vice versa.

Fermion + flux + flux = composite fermion

Wilczek, Jain and many others

Questions

- How does this duality fit in with what we know about 2+1 dimensional dualities?
- Relation to Son's proposal for the ½ filled Landau level (fermionic particle/vortex duality)
- Relation to particle/vortex duality?

One important novelty in 2+1:

In 2+1, every U(1) gauge field gives rise to a “topological” U(1) symmetry:

$$j^{\mu} = \epsilon^{\mu\nu\rho} F_{\nu\rho}$$

Automatically conserved by Bianchi identity. Irrespective of dynamics!

$$\partial_{\mu} j^{\mu} = \epsilon^{\mu\nu\rho} \left(\partial_{\nu} A_{\rho} - \partial_{\rho} A_{\nu} \right) = 0$$

Charge carriers = monopoles.

A zoo of dualities…

Known Duality I: Particle-Vortex Duality for Bosons

Theory A:

$$S = \int d^3x \, |(\partial_\mu - iA_\mu)\phi|^2 - V(\phi)$$

Wilson-Fisher scalar

:Theory B:

$$S = \int d^3x \, |(\partial_\mu - ia_\mu)\Phi|^2 - \tilde{V}(\Phi) + \frac{1}{2\pi}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho$$

Abelian Higgs Model

dynamical gauge field

background gauge field

Boson $\Longleftrightarrow$ **Monopole operator** $\int f = 2\pi$

Son’s duality = “Fermionic” version.

Well established, numerically confirmed

Peskin ’78, Dasgupta and Halperin ‘81

Known Duality II: 3d Bosonization

$$SU(N)_k \text{ coupled to scalars} \leftrightarrow U(k)_{-N+N_f/2,-N+N_f/2} \text{ coupled to fermions},$$

$$U(N)_{k,k} \text{ coupled to scalars} \leftrightarrow SU(k)_{-N+N_f/2} \text{ coupled to fermions},$$

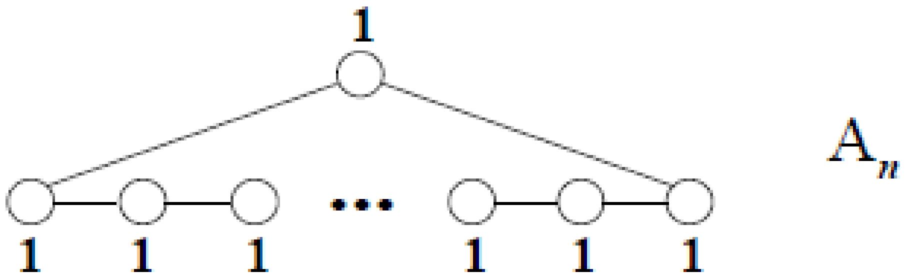
$$U(N)_{k,k+N} \text{ coupled to scalars} \leftrightarrow U(k)_{-N+N_f/2,-N-k+N_f/2} \text{ coupled to fermions}.$$

(Aharony, ….)

- Equivalence between CS gauge theories with bosonic and fermionic matter
- First found at large N with AdS/CFT motivation. Can be checked at large N
- Significant evidence even at finite N:

  self-consistency, level-rank duality after mass deformation, RG flows from SUSY parents
- N=k=Nf=1 reduces to relativistic flux attachment!

Known Duality III: Mirror Symmetry with N=4 SUSY



dual
to

U(1) w/ Nf

(Intriligator, Seiberg 96)

- All Abelian Mirror Pairs follow from basic duality:

U(1) w/ one Hypermultiplet = free Hypermultiplet

(Kapustin, Strassler 99)

- Also comes with non-Abelian version
- SUSY allows for very rigorous tests!

Claim:

All known Abelian non-SUSY dualities (and some new ones) can be derived from one seed duality!

A Seed Duality

Theory A:

(Φ,a dynamical)

$$S_{\text{scalar+flux}} = S_{\text{scalar}}[\phi; a] + S_{CS}[a] + S_{BF}[a; A]$$

Theory B:

(fermion dynamical)

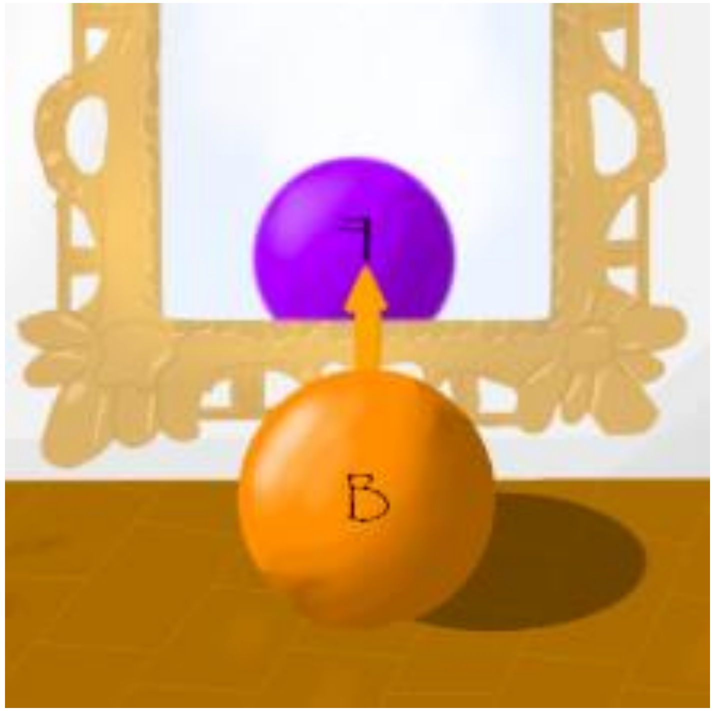
$$S = S_{\text{fermion}}[A] - \frac{1}{2} S_{CS}[A]$$

contact term
ensures Hall conductivities agree

Polyakov ’88 and many others

(More recently, many works on non-Abelian versions of this duality, notably by Minwalla et. al. and Aharony et.al.)

A Seed Duality



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Theory B:

$$S = S_{\text{fermion}}[A] - \frac{1}{2} S_{CS}[A]$$

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}a \; Z_{\text{scalar}}[a] \exp\Big(iS_{CS}[a] + iS_{BF}[a; A] \Big) = Z_{\text{fermion}}[A]\, e^{-\frac{i}{2} S_{CS}[A]}$$

Scalar with flux = Fermion

Basic Idea

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}a \; Z_{\text{scalar}}[a] \exp\left(iS_{CS}[a] + iS_{BF}[a;A] \right) = Z_{\text{fermion}}[A] \, e^{-\frac{i}{2}S_{CS}[A]}$$

Manipulate path integral to derive other dualities

- c.f. Kapustin and Strassler '99 for supersymmetric theories

Can do path integrals over A with Z[A] as the integrand.

Example: Another Flux Attachement

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}a \; Z_{\text{scalar}}[a] \exp\Big(iS_{CS}[a] + iS_{BF}[a;A] \Big) = Z_{\text{fermion}}[A]\, e^{-\frac{i}{2}S_{CS}[A]}$$

Promote *A* to dynamical gauge field. Right-hand side becomes

$$Z_{\text{fermion+flux}}[C] = \int \mathcal{D}A \; Z_{\text{fermion}}[A] \exp\Big(-\frac{i}{2}S_{CS}[A] - iS_{BF}[A;C] \Big)$$

new background field

Do the same on left-hand side.

$$\int \mathcal{D}A \; Z_{\text{scalar+flux}}[A]\, e^{-iS_{BF}[A;C]} = Z_{\text{scalar}}[C]\, e^{iS_{CS}[C]}$$

integrate out *A;* equation of motion
also eliminates *a* through $da = dC$

$$\Longrightarrow \quad \boxed{Z_{\text{fermion+flux}}[C] = Z_{\text{scalar}}[C]\, e^{iS_{CS}[C]}}$$

Chen, Fisher, Wu ‘93

Applications: Known Dualities

Same Procedure Derives:

- Particle-Vortex Duality
- Son’s fermionic Particle-Vortex duality
- Novel self-duality of 2-flavor QED and its duality two 2-flavor scalar QED. (Xu and You ’15)

Gives us significant confidence in the base pair.

Applications: Known Dualities

Note that flux attachment is, roughly, “square root”
of particle/vortex duality.

Can derive P/V duality from flux attachment, but not vice versa.

Evidence for the base pair

- Assuming the base pair, we can derive many known, independently verified dualities
- The base pair is the N=1 case of 3d bosonization, which has been rigorously established at large N and significant evidence exist at finite N
- Supersymmetric Mirror symmetry can be deformed to reduce to the base pair; minimal assumptions on the structure of the phase diagram needed to make the argument.

(Kachru, Mulligan, Toroba, Wang)

Novel Applications

Non-supersymmetric “mirror symmetry”

(AK, Brandon Robinson, David Tong)

Taking our derivation seriously, we can derive many more dual pairs, for example

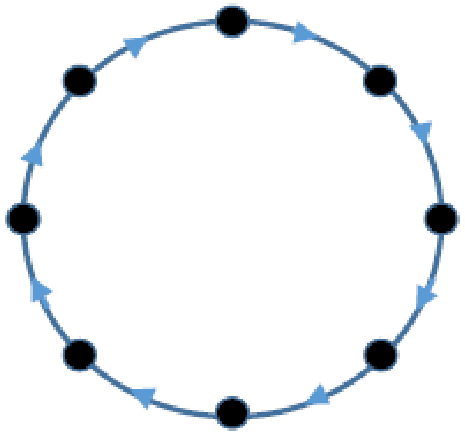
U(1) w/ Nf fermions

dual to

U(1)$^{\text{Nf-1}}$ quiver gauge theory

→ **bifundamental WF scalar**

● **U(1) gauge group**

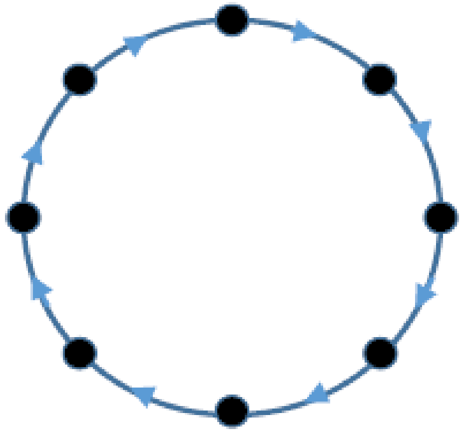


CS - terms

Easiest version: **U(1) w/ Nf fermions, CS=-Nf/2**

- Take Nf free fermions (w/ CS= -1/2), each with its own background gauge field
- Gauge diagonal U(1)

dual
to



- (WF scalar + a + CS=1)^Nf
- gauging diagonal background U(1)
  sets diagonal a to zero.

$$k^{pq} = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & \ddots & & \\ & & & 2 & -1 \\ & & & -1 & 2 \end{pmatrix} .$$

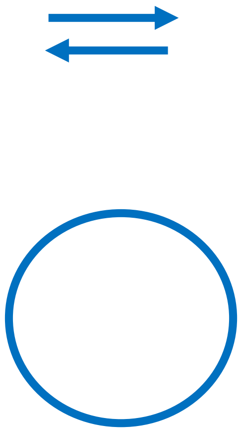
Operator mapping

QED

Quiver

meson

monopole

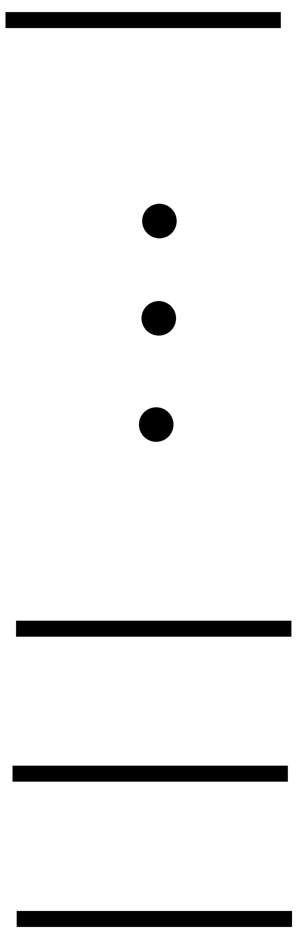


unique at CS=-Nf/2
Nf zero modes, but unique GS
obeying Gauss' law.

Global symmetry charges match. Dimensions? Not easy to calculate on either side.

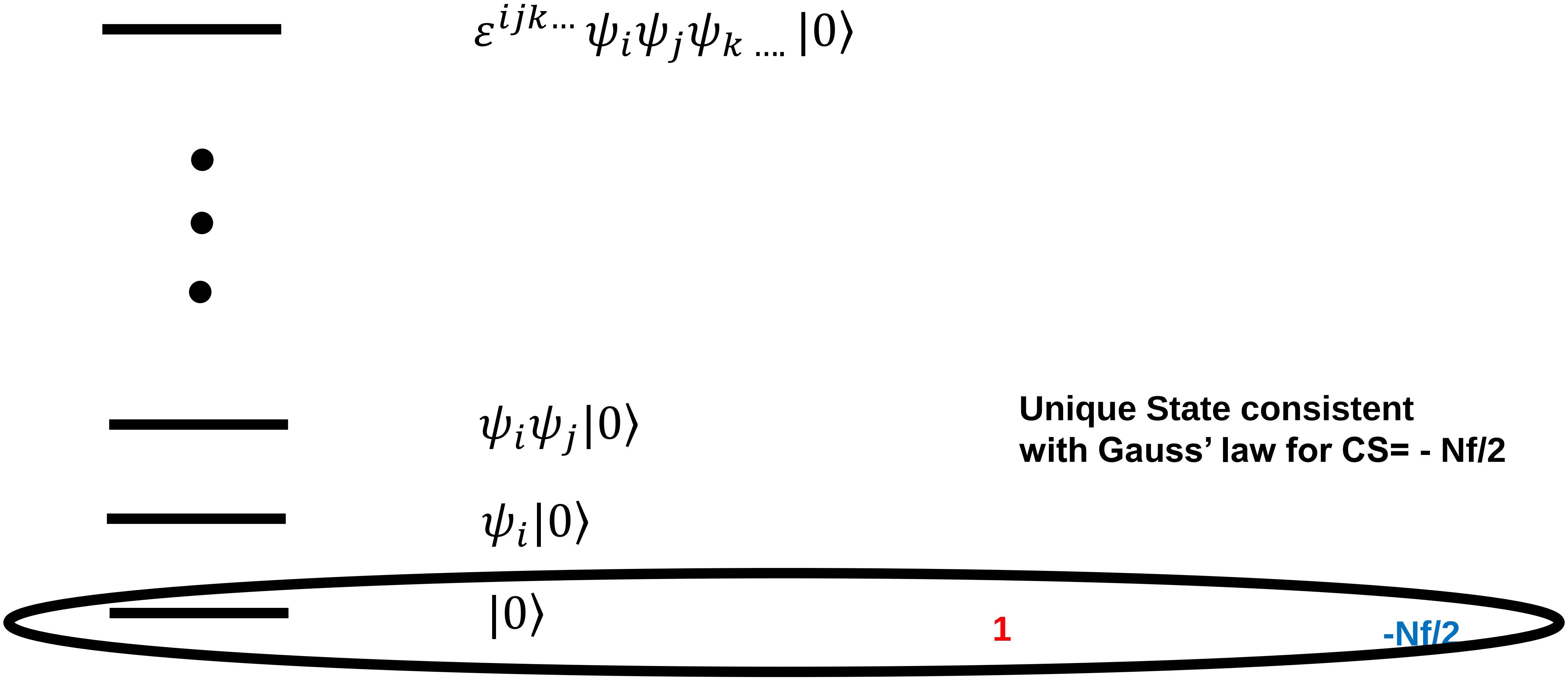
Monopoles Spectrum

(Borokhov, Kapustin, Wu)



	Degeneracy	Charge
$\varepsilon^{ijk\ldots}\psi_i\psi_j\psi_k \ldots\ \lvert 0\rangle$	1	Nf/2
$\psi_j\psi_i\lvert 0\rangle$	Nf (Nf-1)/2	
$\psi_i\lvert 0\rangle$	Nf	-Nf/2 + 1
$\lvert 0\rangle$	1	-Nf/2

Monopoles Spectrum



$\varepsilon^{ijk\dots}\psi_i\psi_j\psi_k \dots\ |0\rangle$

$\psi_i\psi_j|0\rangle$

$\psi_i|0\rangle$

$|0\rangle$

Unique State consistent
with Gauss' law for CS= - Nf/2

1

-Nf/2

Operator mapping

QED

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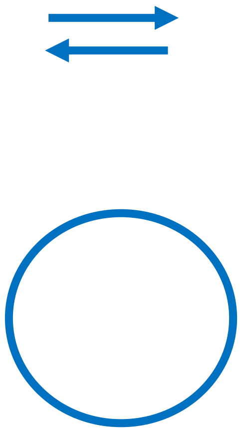
meson

monopole



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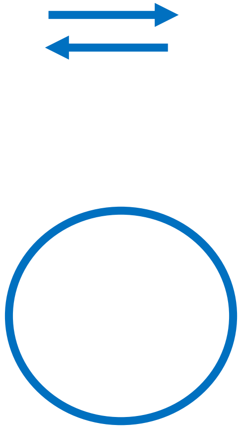


Allows large Nf limit! Solvable.

Δ(monopole)= 0.265 Nf + …

(Borokhov, Kapustin, Wu)

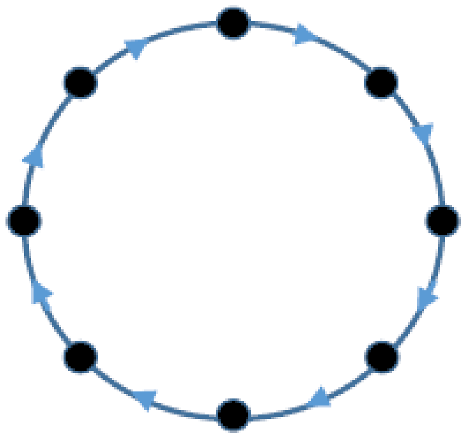
Abelian quiver with N nodes should simplify in large N limit!

CS - terms

More interesting: **U(1) w/ Nf fermions, |CS| < Nf/2**

- Take NL free fermions w/ CS= -1/2 and NR w/ CS = +1/2
- Gauge diagonal U(1)

dual
to

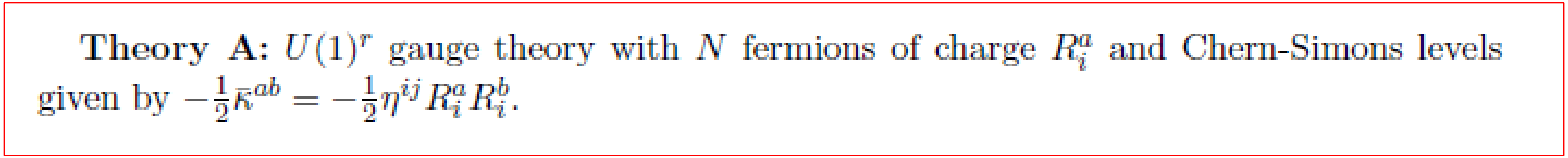


$$\kappa^{pq} = \begin{pmatrix} 2 & -1 & & & & & \\ -1 & \ddots & & & & & \\ & & 2 & -1 & & & \\ & & -1 & 0 & +1 & & \\ & & & +1 & -2 & & \\ & & & & & \ddots & +1 \\ & & & & & +1 & -2 \end{pmatrix},$$

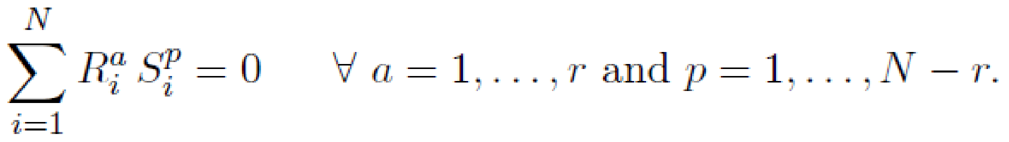
Operator map can still be worked out!

(Melby-Thompson; unpublished)

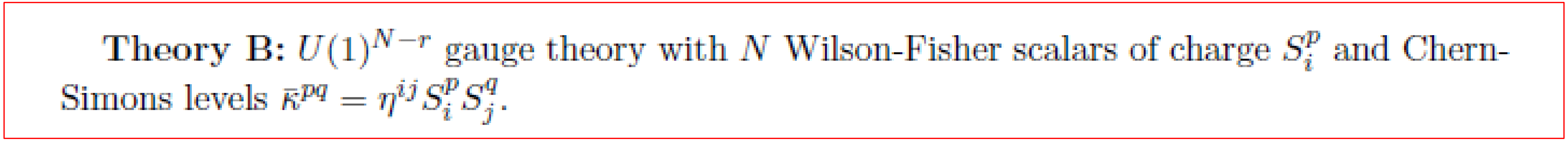
General Abelian Bosonization Duality



Theory A: $U(1)^r$ gauge theory with N fermions of charge R_i^a and Chern-Simons levels given by $-\frac{1}{2}\bar{\kappa}^{ab} = -\frac{1}{2}\eta^{ij}R_i^a R_i^b$.



$$\sum_{i=1}^{N} R_i^a\, S_i^p = 0 \qquad \forall\; a = 1, \ldots, r \text{ and } p = 1, \ldots, N - r.$$



Theory B: $U(1)^{N-r}$ gauge theory with N Wilson-Fisher scalars of charge S_i^p and Chern-Simons levels $\bar{\kappa}^{pq} = \eta^{ij}S_i^p S_j^q$.

Boson/Boson and Fermion/Fermion Dualities

- Two bosonizations = particle/vortex (boson-boson or fermion-fermion)

- 2N bosonizations = U(1) w/ Nf to quiver duality for bosons only / fermions only

- special case: U(1) w/ 2 fermions is self-dual

(Xu/Yu;
rederived using bosonization w/ David Tong and
by Hsin/Seiberg)

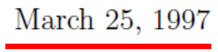
Upshot: Path integral techniques allow to derive a large family of Abelian dualities from conjectured base pair.

Future Projects/Speculations

- 3d to 2d (w/ David Tong and Carl Turner)
- Stringy Realization of 3d bosonization

  (w/ Kristan Jensen)

Duality and Compactification



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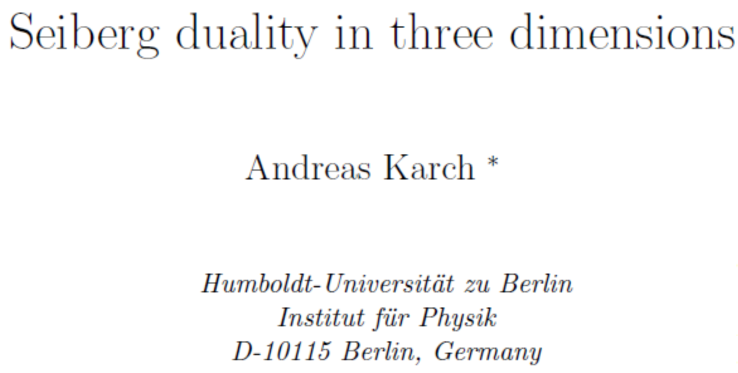

Seiberg duality in three dimensions

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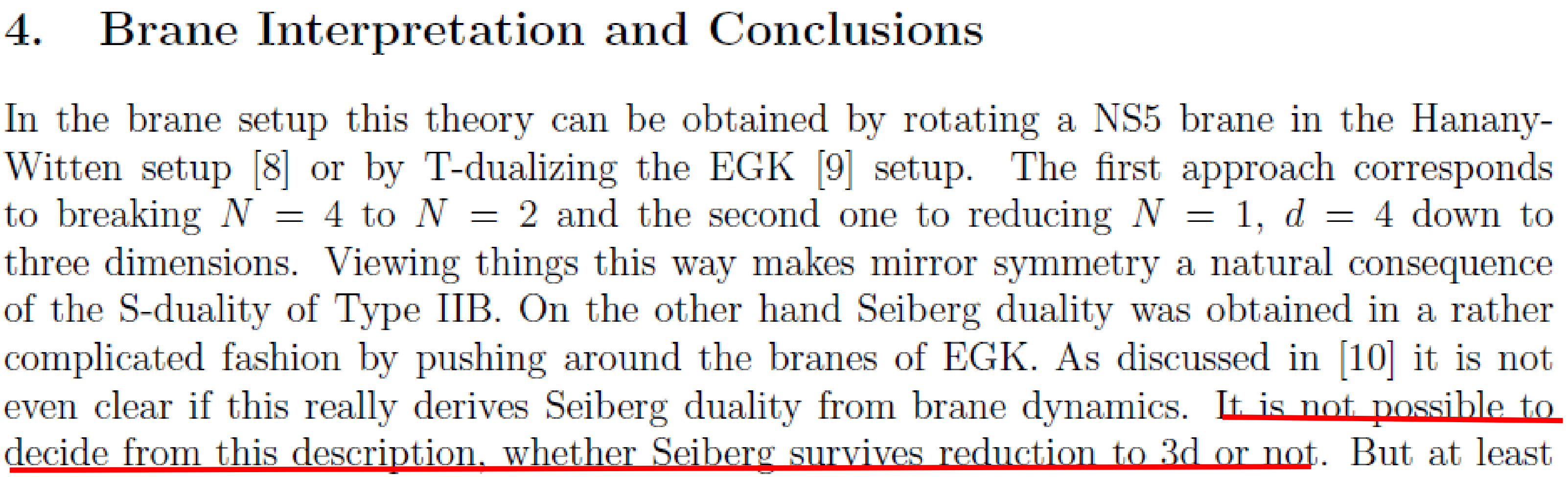
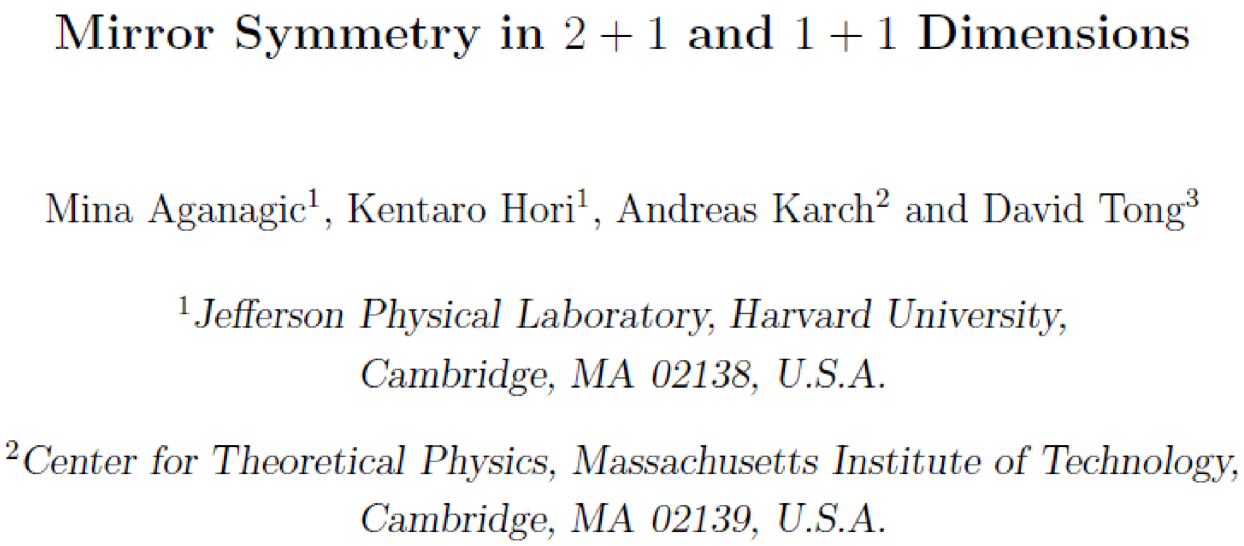
D-10115 Berlin, Germany



4. Brane Interpretation and Conclusions

In the brane setup this theory can be obtained by rotating a NS5 brane in the Hanany-Witten setup [8] or by T-dualizing the EGK [9] setup. The first approach corresponds to breaking $N = 4$ to $N = 2$ and the second one to reducing $N = 1$, $d = 4$ down to three dimensions. Viewing things this way makes mirror symmetry a natural consequence of the S-duality of Type IIB. On the other hand Seiberg duality was obtained in a rather complicated fashion by pushing around the branes of EGK. As discussed in [10] it is not even clear if this really derives Seiberg duality from brane dynamics. It is not possible to decide from this description, whether Seiberg survives reduction to 3d or not. But at least

Duality and Compatification



Mirror Symmetry in $2+1$ and $1+1$ Dimensions

Mina Aganagic[1], Kentaro Hori[1], Andreas Karch[2] and David Tong[3]

[1] *Jefferson Physical Laboratory, Harvard University,*
Cambridge, MA 02138, U.S.A.

[2] *Center for Theoretical Physics, Massachusetts Institute of Technology,*
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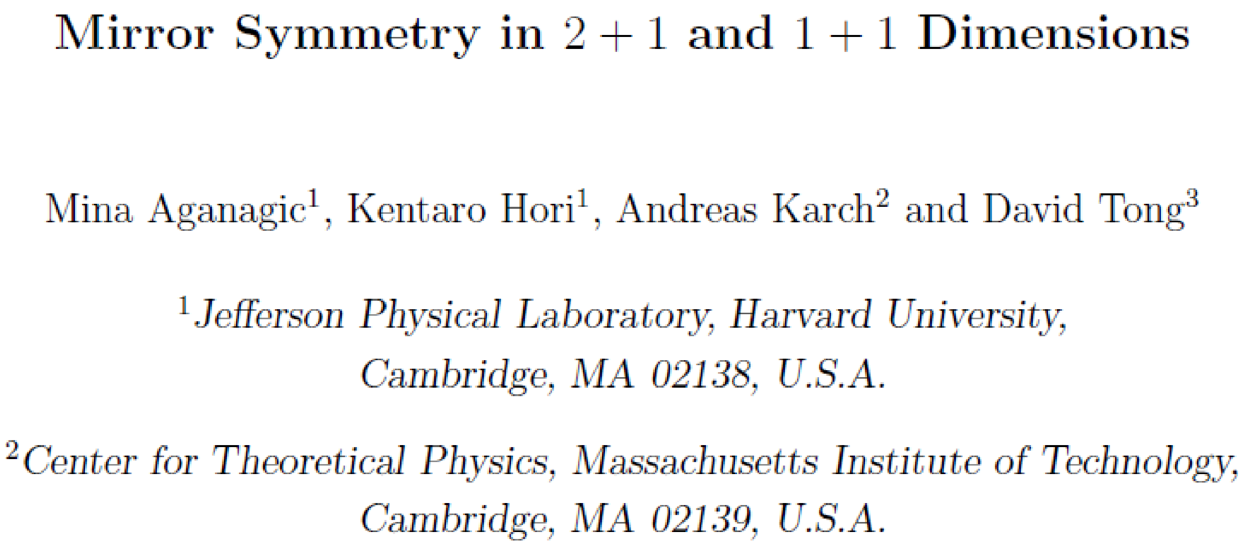
a few years later: 3d to 2d
for SUSY theories.

Mirror symmetry implies
Mirror symmetry

Duality and Compactification



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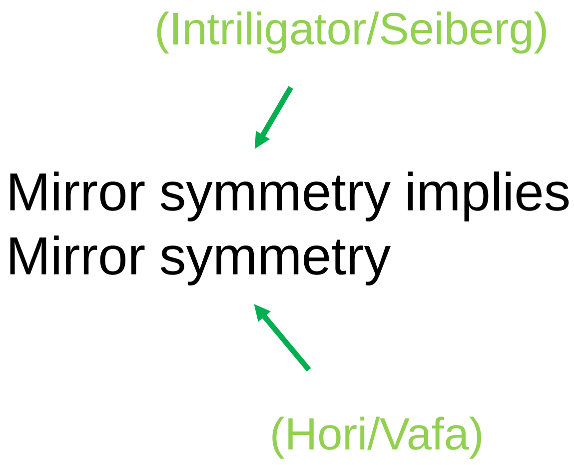


(Intriligator/Seiberg)

Mirror symmetry implies
Mirror symmetry

(Hori/Vafa)

Important parameter: $\gamma = g^2 R$

- γ << 1: Compactify weakly coupled UV theory. Known 2d Lagrangian
- γ >> 1: Flow to IR first. Duality is actually valid.

Non-SUSY Duality and Compactification

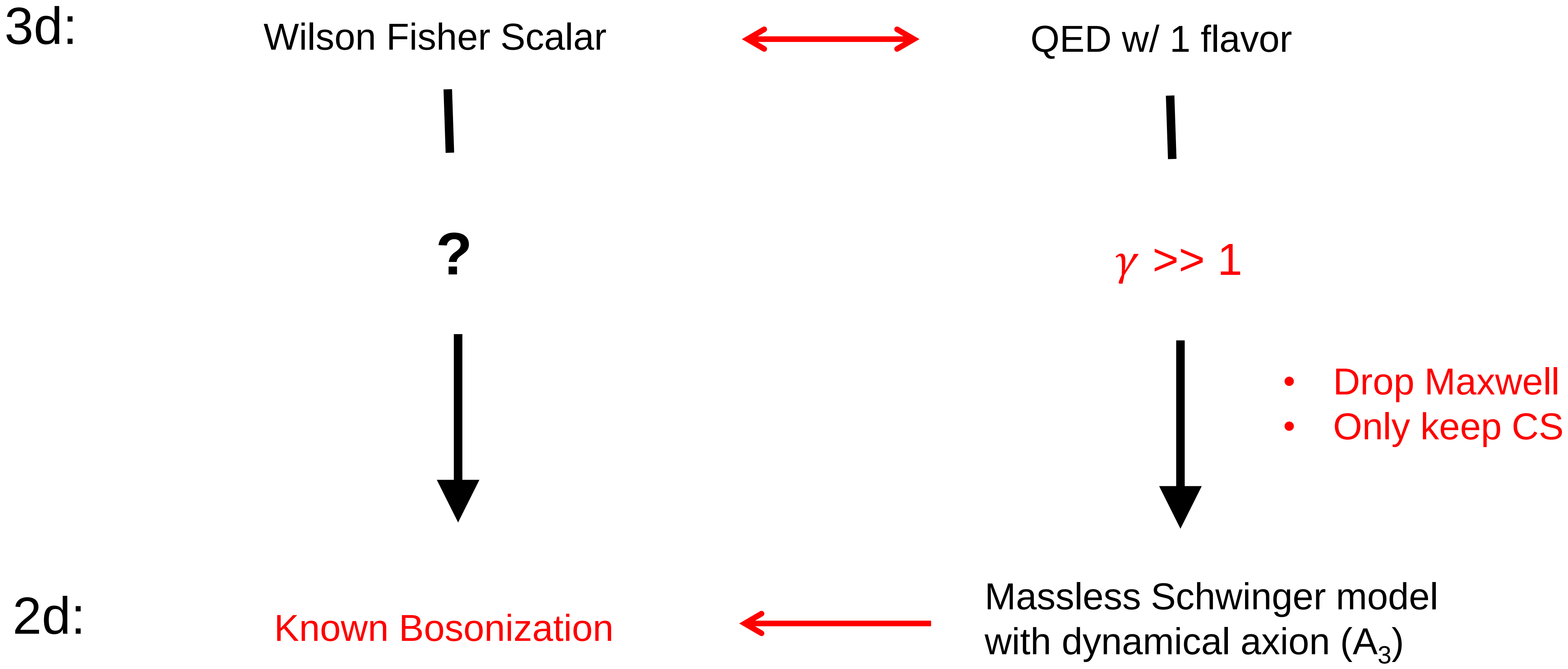
3d: Wilson Fisher Scalar ↔ QED w/ 1 flavor

$\gamma >> 1$ ↓

- Drop Maxwell
- Only keep CS

2d: Known Bosonization ← Massless Schwinger model with dynamical axion (A_3)

Non-SUSY Duality and Compactification



3d:

Wilson Fisher Scalar

QED w/ 1 flavor

?

$\gamma >> 1$

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2d:

Known Bosonization

Massless Schwinger model
with dynamical axion (A_3)

Stringy Realization of 3d Bosonization

$$SU(N)_{-k+N_f/2} \text{ with } N_f \text{ fermions} \quad \leftrightarrow \quad U(k)_N \text{ with } N_f \text{ scalars}$$

Usual limit: N>>1, k>>1, k/N finite.

- effectively a vector model; CS gauge field enforces restriction to singlets
- dual description in terms of a higher spin theory
- this is how the duality was discovered in the first place

Stringy Realization of 3d Bosonization

$$SU(N)_{-k+N_f/2} \text{ with } N_f \text{ fermions} \quad \leftrightarrow \quad U(k)_N \text{ with } N_f \text{ scalars}$$

Claim: nice stringy embedding for: N>>1, k finite, k/N $\to 0$ (probe limit)

Stringy Realization of 3d Bosonization

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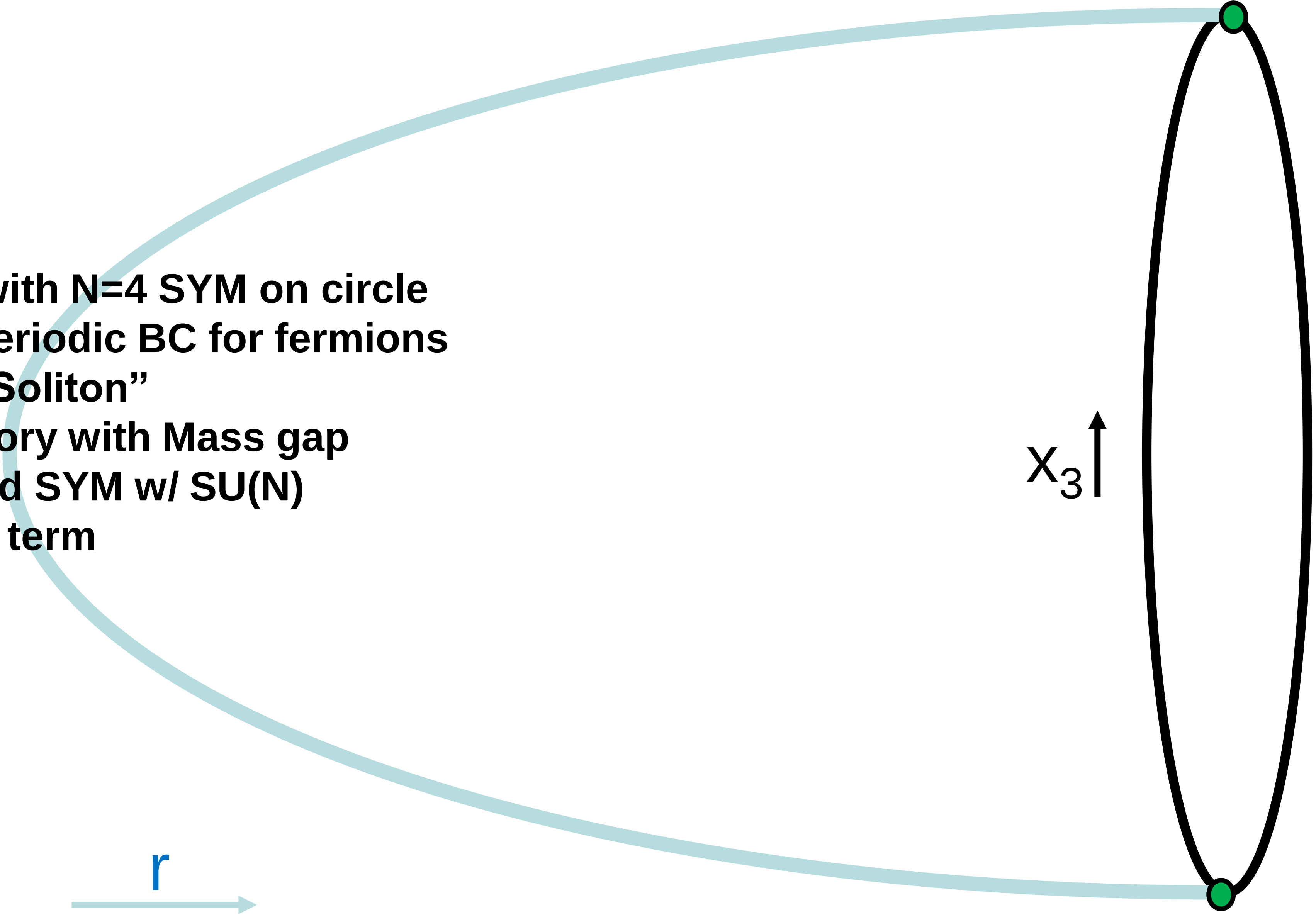
Claim: nice stringy embedding for: N>>1, k finite, k/N $\rightarrow 0$ (probe limit)

- Without flavors theories are gapped, but non-trivial topological FTs
- Bosonization duality reduces to well-known level/rank duality of TFTs
- SU(N) side can be embedded as low energy limit into standard large N gauge theory in a gapped phase
- 3d Bosonization = adding flavor to level/rank duality.

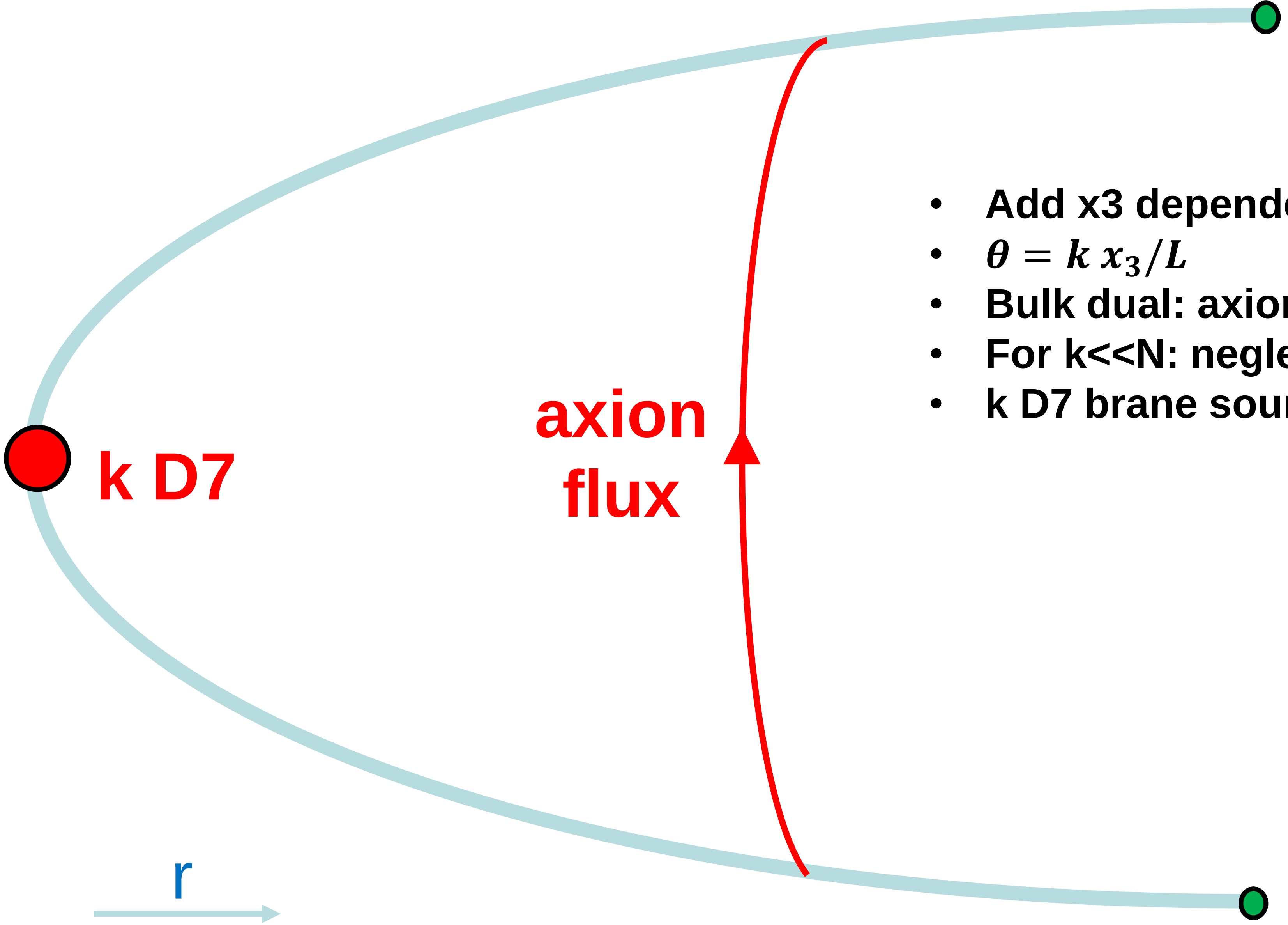
Stringy Realization of $SU(N)_k$

- Start with N=4 SYM on circle
- Anti-periodic BC for fermions
- “AdS-Soliton”
- 3d theory with Mass gap
- Pure 3d SYM w/ SU(N)
- No CS term



(Fujita, Li, Ryu Takayanagi)

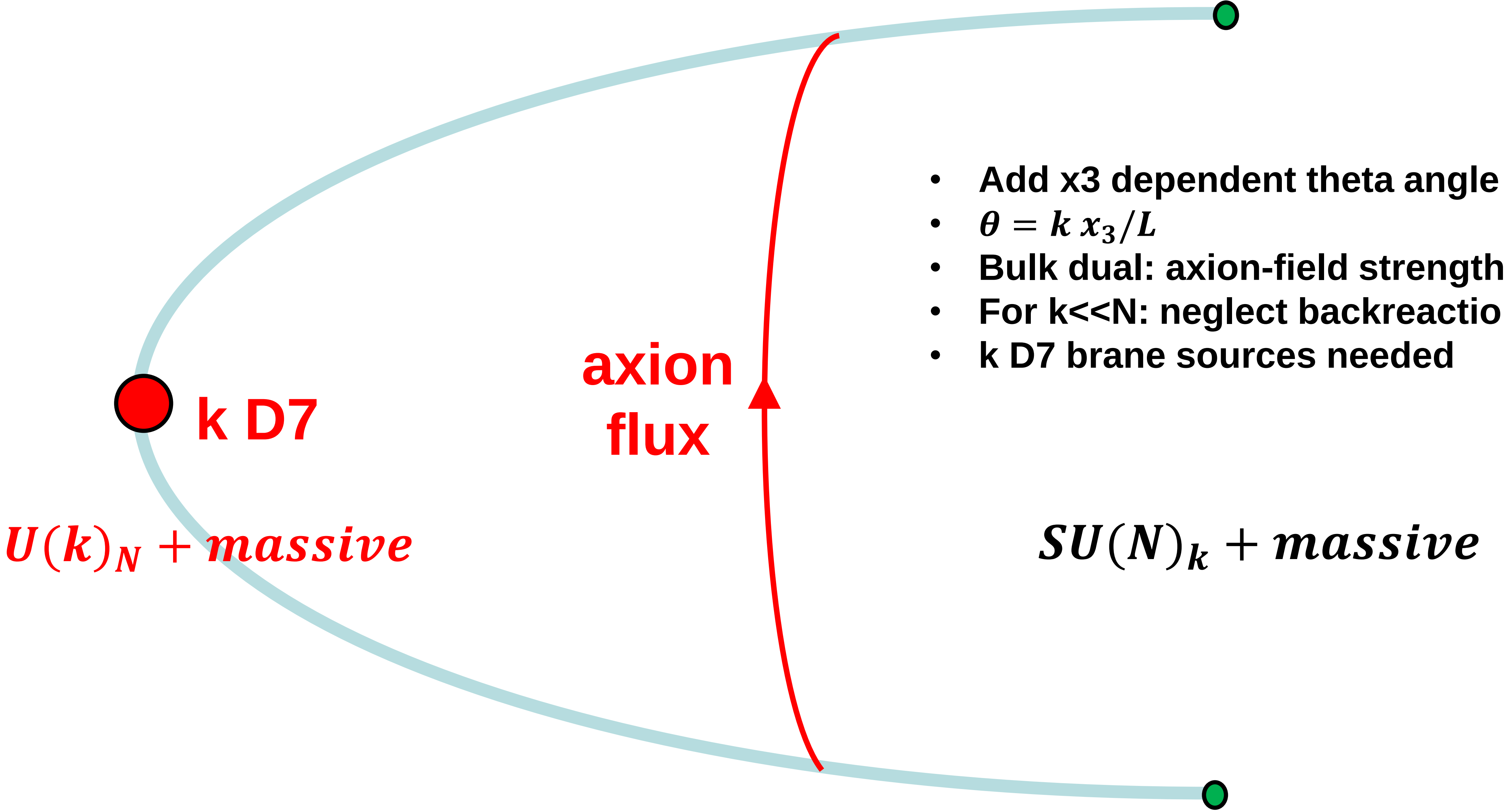
Stringy Realization of SU(N)$_k$



- **Add x3 dependent theta angle**
- $\boldsymbol{\theta = k\, x_3/L}$
- **Bulk dual: axion-field strength**
- **For k<<N: neglect backreaction**
- **k D7 brane sources needed**

(Fujita, Li, Ryu Takayanagi)

Stringy Realization of SU(N)$_k$



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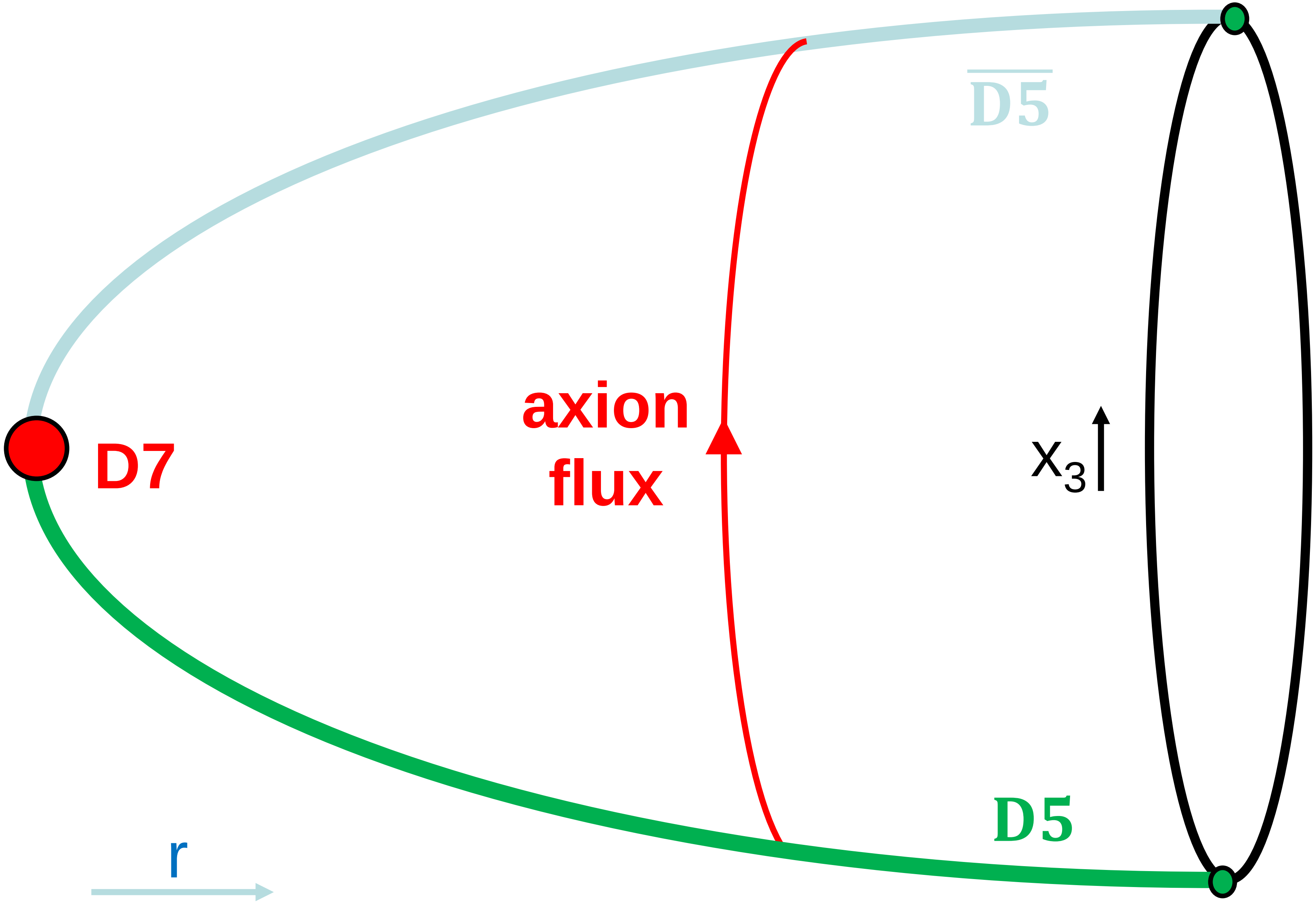
holography=level/rank

(Fujita, Li, Ryu Takayanagi)

Stringy Realization of 3d Bosonization: **Add Flavor!**



Stringy Realization of 3d Bosonization: **Add Flavor!**

- Bulk: 4d hypermultiplet on S^3 x R
- No fermion zero mode on sphere.
- Only scalars light, charged fermions heavy

$U(k)_N$+ scalars

- Boundary: all scalars pick up masses from loops
- Some light fermions may remain protected by symmetry
- Question: How many?

$SU(N)_k$+ fermions

Conclusions

- Powerful new tools to study dualities in 2+1 dimensions
- Many problems still open:
    - Reduction to 2d
    - Stringy Embedding
    - Applications to Quantum Hall Physics?