Composite Higgs Approach to BSM

2. Five-dimensional models of Composite Higgs

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March 2014
In the previous lecture, I listed 3 strategies for building models with calculable Higgs potential:

supersymmetry, extra dimensions, Goldstone bosons

I discussed the third of these, now it is time to discuss the second.

I will consider here models with one additional space dimensions, either flat or “warped” -- that is, a section of AdS, following Randall-Sundrum.
The idea of a warped extra dimension is related to the idea of new strong coupling dynamics through Maldacena’s AdS/CFT duality.

The strong coupling dynamics is represented as weak-coupling dynamics in a space with one extra dimension. Depth in the extra dimension represents length scale in the original 4-dimensional description.

In this discussion, I will use this idea mainly as a motivation for some parts of the construction. And, before going to AdS, I would like to do some analysis in flat space.
Even in flat 5-d space, there are analogies to the models of the previous lecture.

To introduce the basic concepts, consider first a scalar field in 5 flat dimensions, with the 5th dimension represented by a circle of circumference $2\pi R$. $M = 0, 1, 2, 3, 5$

Klein-Gordon equation:

$$(\partial_M \partial^M) \phi(x) = 0$$

Fourier analysis:

$$\phi(x^M) = \sum_n \phi_n(x^\mu) e^{inx^5}/R$$

The component fields obey the equations

$$(\partial_\mu \partial^\mu + \frac{n^2}{R^2}) \phi_n(x) = 0$$

We find series of complex scalar fields with increasing mass. This is the Kaluza-Klein tower of states.
With periodic boundary conditions, the spectrum is very simple. To add complexity, it is useful to consider orbifold boundary conditions. Here is the simplest example: Identify \( x^5 \leftrightarrow -x^5 \). This folds the space into a line with two endpoints.

\[ \begin{array}{c}
0 \\
\pi R
\end{array} \]

A field with boundary conditions

\[
\phi(-\epsilon) = \phi(\epsilon), \quad \phi(\pi R - \epsilon) = \phi(\pi R + \epsilon)
\]

has half the number of modes, but the mode

\[
\phi(x^5) = \text{const}
\]

representing a massless particle is still present. A field with the boundary condition

\[
\phi(-\epsilon) = -\phi(\epsilon), \quad \phi(\pi R - \epsilon) = \phi(\pi R + \epsilon)
\]

has half the number of modes, and the massless particle is absent.
Orbifold boundary conditions can be used to model spontaneous symmetry breaking. Consider an set of scalar fields in the adjoint representation of $SU(3)$:

$$\Phi = \begin{pmatrix} A_{ab} & \phi_a^\dagger \\ \phi_a & b \end{pmatrix} \quad a, b = 1, 2$$

Assign the boundary conditions at $x^5 = 0$:

$$\begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

Then there are four massless fields, corresponding to

$$SU(3) \rightarrow SU(2) \times U(1)$$

The remaining fields are present in the model, but only as higher-mass KK states.
This becomes more interesting when we apply it to fields with spin. Consider, for example, a massless fermion in 5-d: $i\gamma \cdot \partial \Psi = 0$

Here

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \gamma^5 = i \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

so the 4-component Dirac field does not split into two 2-component fields as happens in 4-d.

Now do Fourier reduction as before:

$$\Psi(x^M) = \sum \Psi_n(x^\mu) e^{inx^5/R}$$

The mode $n = 0$ splits into two independent fields.

$$\Psi_0 = \begin{pmatrix} -i\psi_L \\ \psi_R \end{pmatrix} \quad i\bar{\sigma} \cdot \partial \psi_L = 0, \ i\sigma \cdot \partial \psi_R = 0$$
A nonzero Fourier mode obeys

\[(i\gamma^\mu \partial_\mu + \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \frac{n}{R})\Psi_n(x^\mu) = 0\]

If we write

\[\Psi_n = \begin{pmatrix} -i\psi_{nL} \\ \psi_{nR} \end{pmatrix}\]

this is equivalent to

\[i\bar{\sigma} \cdot \partial \psi_L + \frac{n}{R} \psi_R = 0 \quad i\sigma \cdot \partial \psi_R + \frac{n}{R} \psi_L = 0\]

which is the usual 4-component Dirac equation,

\[(i\gamma \cdot \partial + m)\psi_n = 0\]

with \(m = n/R\).
An orbifold boundary condition should contain the action of Parity on the fermion field. So we can impose

$$\Psi_0(-\epsilon) = \pm \Gamma \Psi_0(\epsilon) \quad \Gamma = \begin{pmatrix} -1 & \\
& 1 \end{pmatrix}$$

For the choice $-\Gamma$, the massless R fermion is removed, but the massless L fermion remains.

We then have a spectrum with one massless chiral fermion and an infinite tower of massive vectorlike fermions.
The same logic applies to vector fields. A massless vector is accompanied by a KK tower of massive vector fields. We might say that the nth KK state acquires mass through the Higgs mechanism, where the eaten Goldstone boson is 

\[ A^5_n(x) \]

Using the orbifold boundary conditions as for scalar fields, we can model spontaneous breaking of the gauge symmetry. Consider, for example, an SU(3) gauge field. A boundary condition of the form \textit{Parity x (internal)} is

\[ A^\mu : \begin{pmatrix} + & - \\ - & + \end{pmatrix} \quad A^5 : \begin{pmatrix} - & + \\ + & - \end{pmatrix} \]

Then the surviving n=0 states are \text{SU}(2)\times\text{U}(1) gauge bosons and “Higgs” doublet of scalar fields.
There is one more interesting aspect to this theory.

Consider a 5-d fermion charged under a U(1) vector field. If \( A_0^5(x) \neq 0 \) we can remove it by a gauge transformation that takes

\[
\Psi(x^\mu, x^5) \rightarrow e^{igA_0^5 x^5} \Psi(x^\mu, x^5)
\]

However, this rephasing is nontrivial if we are on a periodic space. It corresponds to a change in the boundary condition for \( \Psi(x^\mu, x^5) \) around the periodic direction.
If we integrate out the field $\Psi(x^\mu, x^5)$, there is an effect called the **Hosotani mechanism**: The functional integral over $\Psi$ depends on this phase and is maximized when the phase is (-1), corresponding to the functional description of the Fermi-Dirac thermal ensemble.

We can view this as an induced potential for $A_0^5$ that has a local maximum at $A_0^5 = 0$ and drives the formation of a nonzero v.e.v.
We now have all of the ingredients needed for a model of electroweak symmetry breaking.

Models with this general structure are said to have “Gauge-Higgs unification”. The original model of this type was constructed by Hall, Nomura, and Tucker-Smith. More recently, these modes have been studied intensely by Haba, Hosotani, Lim, and others.
Issues of model-building with Gauge-Higgs unification are similar to those with Little Higgs: We have a mechanism for symmetry-breaking through the Hosotani mechanism, but it typically leads to

\[ v \sim 1/R \]

unless there is a special mechanism to produce a Higgs quartic.

Yukawa couplings arise from gauge couplings in this framework. It is not so difficult to generate the large top quark Yukawa coupling, but it is not so clear how to generate the hierarchy of quark and lepton masses.
Now we can go from flat space to warped space, choosing the 5-d background to be anti-de Sitter
\[ ds^2 = e^{-2ky} \eta_{\mu \nu} dx^\mu dx^\nu - dy^2 \]
on an interval from \( y = 0 \) to \( y = \pi R \).

Randall and Sundrum originally introduced this as a solution to the gauge hierarchy problem. A scalar field in this background has the action
\[
\int dy d^4 x \left[ e^{-2ky} \eta^{\mu \nu} \partial_\mu \phi^\dagger \partial_\nu \phi - e^{-4ky} m^2 |\phi|^2 \right]
\]
For the dynamics near \( y = y_0 \), we should canonically normalize \( \phi : \phi' = e^{-ky} \phi \)

Then the above becomes:
\[
\int dy d^4 x \left[ \eta^{\mu \nu} \partial_\mu \phi'^\dagger \partial_\nu \phi' - e^{-2ky} m^2 |\phi'|^2 \right]
\]
The natural energy scale changes

\[ m \to e^{-ky} m \]

and this decreases exponentially with \( y \). In an AdS/CFT duality picture, the region of the 5-d geometry near \( y = y \) represents the dynamics of the corresponding 4-d theory for momentum scale

\[ Q \sim Q_0 e^{-ky} \]

Randall and Sundrum imagined that we could control the renomalization of the Higgs mass by putting the Higgs field at \( y = \pi R \). However, this construction also opens many more dynamical possibilities.
Here is a mnemonic picture of the 5d space:
Solutions of the scalar field wave equation have the form

\[ \phi(x^\mu, y) = \phi_n(x^\mu) f_n(y) \]

where \( \phi_n \) is a 4-d field satisfying a massive wave equation

\[ (-\partial^2 - m_n^2)\phi_n = 0 \]

and

\[ f_n \sim J_\nu \left( \frac{m_n}{k} e^{k y} \right) + b_{n\nu} Y_\nu \left( \frac{m_n}{k} e^{k y} \right) \]

with

\[ \nu^2 = 4 + \frac{m^2}{k^2} \]

For vector fields, there is a similar structure with

\[ \nu^2 = 1 + \frac{m^2}{k^2} \]
The Randall-Sundrum theory contains an enormous hierarchy of scales, with
\[ e^{k \pi R} \sim \frac{m_{\text{Pl}}}{m_W} \]

We are interested here only in dynamics at the weak-interaction scale, so set
\[ m_n = m_n e^{-k \pi R} \]

As \( y \to 0 \),
\[ Y_\nu \left( \frac{m_n}{k} e^{k(y-\pi R)} \right) \to (e^{k \pi R})^\nu \]

Then \( b_{n\nu} \approx 0 \) and the wavefunctions take the form
\[ f_n \sim J_\nu \left( \frac{m_n}{k} e^{k(y-\pi R)} \right) \]

These move systematically toward smaller \( y \) as the mass level increases.
Agashe, Contino, and Pomarol have proposed a simple model of the Higgs field that fit into this general scheme. They postulate a RS 5-d space with the gauge symmetry

\[ SU(3) \times SO(5) \times U(1)_{B-L} \]

Orbifold boundary conditions on the UV brane at \( y = 0 \) remove gauge field zero modes and break the symmetry to

\[ SU(3) \times SU(2) \times U(1) \]

Orbifold boundary conditions on the IR brane at \( y = \pi R \) preserve

\[ SU(3) \times SO(4) \times U(1)_{B-L} \]

The theory then has global symmetries \( SU(2) \times U(1)_{B-L} \) of which the first factor is the custodial \( SO(3) \). The Higgs bosons are components of \( A^{5A} \) in the coset space \( SO(5)/SO(4) \).
Next consider fermions in the RS geometry. Fermions are intrinsically Dirac, so include a mass term:

$$\int dyd^4x \sqrt{g}[\overline{\Psi}(i\gamma^M D_M - m)\Psi]$$

After the change of variables $\Psi = e^{2ky}\Psi'$, this becomes

$$\int dyd^4x \ e^{ky}\overline{\Psi}' [i\gamma^\mu \partial_\mu - \Gamma \partial_5 - m]\Psi'$$

The Dirac equation has chiral zero modes satisfying

$$[-\Gamma \partial_5 - m] f(y) = 0$$

where

$$\Gamma = \begin{cases} +1 & \text{R} \\ -1 & \text{L} \end{cases}$$

These are of the form

$$f(y) = e^{-k\Gamma cy}$$

where $c = m/k$

At orbifold boundary condition will remove the zero mode of one chirality and leave the other.
The fermion-Higgs interaction comes from
\[ \int dy d^4 x e^{ky} \overline{\Psi}_a [\Gamma (\partial_5 \delta_{ab} - g A^5 t_{ab})] \Psi'_b \]
If \( t_{ab} \) connects fields that have L and R zero modes, the \( y \)-dependence of the integrand is
\[ e^{ky(1/2-c_L)+(1/2-c_R)} A^5(y) \]
Then the mass term of each 5-d fermion multiplet determines the size of the effective Yukawa coupling for its zero mode.
In the dual interpretation, \( c \) is a parameter that models the dynamics of the quark or lepton in the 4-d theory.

A Higgs field expectation value \( A^5(y) \) should be induced at large \( y \).

Fermions with \( c < \frac{1}{2} \) have zero modes also localized a larger \( y \). These have larger Yukawa couplings, from better overlap with the Higgs. We might also regard them as “more composite”.

Fermions with \( c > \frac{1}{2} \) have zero modes localized at smaller \( y \). These have smaller Yukawa couplings, from limited overlap with the Higgs. We might also regard them as “more elementary” or “pointlike”.
A particular issue is obtaining a sufficiently large top quark mass. The \((t, b)_L\) doublet must be quite elementary to satisfy the precision electroweak constraints on \(b_L\), if we do not included exotic fermions. This pushes \(t_R\) to be highly composite.

A consequence of this is that the \(Zt_R\bar{t}_R\) coupling can be modified, by as much as 10\% in some models. This might be visible at the LHC (and measurable precisely at the ILC).
KK excitations of gauge fields in the 5-d bulk can couple to $q\bar{q}$ and appear as resonances in Drell-Yan.

An important point is that the coupling to light fermions is not the standard gauge coupling, but, instead depends on the form of the wavefunctions in the 5th dimension.
The constraints are easiest to understand with flat extra dimensions. The $q\bar{q}$ current is a zero mode. KK excitations have wavefunctions that are orthogonal to this zero mode; at this level, the Drell-Yan production is zero.

A parity symmetry in the extra dimensions can make the amplitude for $q\bar{q} \rightarrow Z'_1$ exactly zero. Typically, there are new operators at the boundaries of the extra dimension that can affect the form of the KK wavefunctions.

$$\epsilon \int d^4x (F_{\mu\nu})^2 \big|_{x^5=0,\pi R} \quad \epsilon \sim 0.1$$

These can give suppressed but nonzero rates for $q\bar{q} \rightarrow Z'_2 \rightarrow \mu^+ \mu^-$ proportional to $\epsilon^4$. 
In warped models, the Drell-Yan cross sections can be suppressed or enhanced, depending on the detailed form of the wavefunctions, which is model-dependent.

Here is an example from a warped 5-d model of Davoudiasl, Hewett, and Rizzo.

\[ \nu = \sqrt{1 + \frac{m_{KK}^2}{k^2}} \]
A final question: Is the gluon in the bulk (and, thus, partially composite) or is it an elementary particle?

If the latter, there are KK gluon states at the same (few TeV) mass scale as the other KK excitations.

KK gluons decay dominantly to the most composite quarks, \( q\bar{q} \rightarrow Z_g \rightarrow t_R\bar{t}_L \).

The search for this signature (Randall, Wang, Agashe, and others) provided some of the motivation for the development of jet-substructure top tagging.
A 5-dimensional model can then provide a realization of the Composite Higgs story - or, in fact, a dual realization.

Vectorlike top, W, Z partners are automatically present at KK excitations.

Symmetry breaking can be driven by the Hosotani mechanism.

Flavor-dependent Yukawa couplings can be explained by mixing of quarks and leptons with strongly interacting states. In warped compactifications, this mixing has a geometrical representation.