AdS Amplitudes &
The Definition of Holographic CFT

Eric Perlmutter, Caltech

20 Years Later: The Many Faces of AdS/CFT

November 3, 2017
The Large N Limit of Superconformal Field Theories and Supergravity

Juan M. Maldacena

(Submitted on 27 Nov 1997 (v1), last revised 22 Jan 1998 (this version, v3))

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8. The Large N limit of superconformal field theories and supergravity

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Existential Questions for AdS/CFT

What constraints on CFT microscopics are responsible for the emergence of a weakly coupled bulk description?

What are the allowed couplings among light degrees of freedom?
Can we discover the necessity of string/M-theory from CFT?
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What is the organizing principle underlying the structure of scattering amplitudes in AdS, and the 1/N expansion of CFTs?
• This talk is based on work from 2016/17 with O. Aharony, F. Alday, A. Bissi, and WIP with D. Meltzer.
Holography from Conformal Field Theory

Idse Heemskerk\(^1\), Joao Penedones\(^2\), Joseph Polchinski\(^2\), James Sully\(^1\)

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\(^2\) Kavli Institute for Theoretical Physics, Santa Barbara, California 93106-4030, USA
HPPS

• 1-to-1 map between solutions of large N crossing and AdS contact interactions.

\[ \mathcal{O} \times \mathcal{O} \sim 1 + \sum_{n, \ell} [\mathcal{O} \mathcal{O}]_{n, \ell} + \cdots \]

• At infinite N, this is generalized free scalar field theory. At O(1/N^2), they found solutions of bounded spin: \( \gamma_{n, \ell} > L = 0 \). There are \( \frac{(L+2)(L+4)}{8} \) solutions: precisely one for every independent, local quartic bulk vertex with \( \leq 2L+2 \) derivatives.
AdS loops and trees from CFT crossing symmetry

We can add cubic couplings to this story. This reconstructs AdS exchange amplitudes.

\[ \mathcal{O} \times \mathcal{O} \sim 1 + \sum_{n,\ell} [\mathcal{O}\mathcal{O}]_{n,\ell} + \frac{1}{N} \mathcal{O}_{\tau,s} + \cdots \]  

[Alday, Bissi, EP]

More interesting is the question of loops in AdS. Very few computations:

Yes:

\[
\begin{align*}
&\quad \\
\end{align*}
\]

No:

\[
\begin{align*}
&\quad \\
\end{align*}
\]

A gaping hole in our understanding of AdS amplitudes!

[Penedones; Fitzpatrick, Kaplan; Aharony, Alday, Bissi, EP]
Why else to study AdS loops?

1. **The structure of large $N$**
   - What defines a holographic CFT at subleading orders in $1/N$?
     
     \[
     \mathcal{G}(u, v) = \mathcal{G}^{(0)}(u, v) + \frac{1}{N^2} \mathcal{G}^{(1)}(u, v) + \frac{1}{N^4} \mathcal{G}^{(2)}(u, v) + \cdots
     \]

2. **Amplitudes in curved space**
   - Flat space loop amplitudes are extremely rich. What happens in curved space?
   - AdS amplitudes admit flat space limit $\rightarrow$ Flat space structures should be hiding in AdS.

3. **Non-planar corrections to specific CFTs**
   - Even in a highly symmetric theory like 4d $N=4$ Super-Yang-Mills, I know of no other method, even in principle, for computing non-planar quantities at strong coupling. (e.g. integrability fails beyond planar limit.)
Solving 1-loop crossing

Following HPPS, solve 1-loop crossing in perturbation around generalized free fields.

Basic idea:

1. $\mathcal{G}_{1\text{-loop}}$ contains the following term at $v \ll 1$, where $f(u)$ is fixed by tree-level data:

$$\mathcal{G}_{1\text{-loop}}(u, v) \supset u^\Delta f(u) \log^2 u \log v \quad (\gamma_{n,\ell}^{(1)})^2 \text{ terms}$$
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\[ \sum_{\mathcal{O}} \mathcal{O} = \sum_{\mathcal{O}'} \mathcal{O}' \]

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2. By crossing, this implies the existence of a term

\[ \mathcal{G}_{1\text{-loop}}(u, v) \ni u^\Delta f(v) \log^2 v \log u \]

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Solving 1-loop crossing

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\[ \mathcal{G}_{1\text{-loop}}(u, v) \supset u^\Delta f(v) \log^2 v \log u \quad \gamma_{0,\ell}^{(2)} \text{ terms} \]

3. This gives an equation for \( \gamma_{0,\ell}^{(2)} \) in terms of \( \gamma_{n,\ell}^{(1)} \).
Solving 1-loop crossing

Following HPPS, solve 1-loop crossing in perturbation around generalized free fields.

\[ \sum_{\mathcal{O}} \mathcal{O} = \sum_{\mathcal{O'}} \mathcal{O'} \]

\[ v^\Delta g_{1\text{-loop}}(u, v) = u^\Delta g_{1\text{-loop}}(v, u) \]

• Amounts to solving for the double-trace anomalous dimensions at \( \mathcal{O}(1/N^4) \).

• Solve in large spin expansion

\[ \gamma_{0, \ell}^{(2)} = \frac{1}{\ell^{2\Delta}} \left( b_0 + \frac{b_1}{\ell^2} + \frac{b_2}{\ell^4} + \frac{b_3}{\ell^6} + \cdots \right) \]

• A nice class of harmonic polylogs forms a basis of solutions.

• Bulk divergences are nicely manifest in CFT:

\[ \text{AdS divergences} = \text{Low-spin divergences in OPE data} \]

This follows from the locality of bulk counterterms.
Example: One-loop triangle in $\phi^3 + \phi^4$

$$\mathcal{L}_{\text{bulk}} = \frac{1}{2}(\partial \phi)^2 + (2 - d)\phi^2 + \frac{\mu_3}{3!}\phi^3 + \frac{\mu_4}{4!}\phi^4$$

- Computations are most transparent in Mellin space.
- In $\text{AdS}_5$, 

$$M_{1-\text{loop}}(s,t) = \left( \frac{40}{t-4} {\frac{3F_2}{t-4} \left( 1, 1, 2 - \frac{t}{2}, \frac{5}{2}, 3 - \frac{t}{2}; 1 \right)} + \frac{56}{5} {\frac{3F_2}{t-6} \left( 2, 2, 3 - \frac{t}{2}, \frac{7}{2}, 4 - \frac{t}{2}; 1 \right)} \right) \mu_3^2 \mu_4$$

- In $\text{AdS}_3$, 

$$M_{1-\text{loop}}(s,t) = \left( \frac{6(\psi(2 - \frac{t}{2}) + \gamma)}{t-2} - \frac{3F_2}{t-4} \left( 1, 1, 2 - \frac{t}{2}, \frac{5}{2}, 3 - \frac{t}{2}; 1 \right) \right) \mu_3^2 \mu_4$$
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- In AdS$_5$,

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- In AdS$_3$,

$$M_{1\text{-loop}}(s, t) = \left( \frac{6 (\psi \left( 2 - \frac{t}{2} \right) + \gamma)}{t - 2} - \frac{3F_2 \left( 1, 1, 2 - \frac{t}{2}; \frac{5}{2}, 3 - \frac{t}{2}; 1 \right)}{t - 4} \right) \mu_3^2 \mu_4$$

No diagrams necessary!

(See more recent work for diagrammatic progress:) [Cardona; Yuan; Giombi, Sleight, Taronna]
Now let's pivot to a long-standing existential question: the meaning of “holographic CFT”.

**Q1:** What are the sufficient conditions for a CFT to have a weakly coupled, local, Einstein gravity dual?

**Q2:** Given some properties of a family of CFTs in the large N limit, what does the bulk dual look like?
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Closing the gap: A brief history

HPPS (2009):

**Large N + Large gap** to higher-spin single-trace operators = Weakly coupled, local gravity dual

All CFT data becomes “strongly coupled” due to a single spectral condition.
Adding cubic interactions doesn't spoil locality...But what *kind* of local theory?

\[ \Delta_{\text{gap}} \sim M_{HS} \]

CEMZ (2014): Prototypical holographic CFT should have a local **Einstein** gravity dual.

\[ \langle TTT \rangle \sim \langle TTT \rangle_{\text{Einstein}} + \Delta_{\text{gap}}^{-2} \langle TTT \rangle_{R^2} + \Delta_{\text{gap}}^{-4} \langle TTT \rangle_{R^3} \]

4d CFT:

\[ \frac{|a - c|}{c} \leq \Delta_{\text{gap}}^{-2} \]

[See also Afkhami-Jeddi, Hartman, Kundu, Tajdini; Costa, Hansen, Penedones]

Caron-Huot (2017): higher-derivative \(\#(\partial) > 2\) quartic vertices, generated by integrating out heavy particles, are suppressed by \(\Delta_{\text{gap}}\) (with some exceptions, e.g. \((\partial \phi)^4\))
Beyond $a=c$

The $a$-$c$ bound is somewhat unsatisfactory.

1) In SCFTs, neither $a$ nor $c$ is a function of $\Delta_{\text{gap}}$. Instead, $a$-$c$ obeys a stronger bound

\[
\frac{|a - c|}{c} \lesssim \frac{1}{\Delta_{\text{gap}}^2}
\]

where $\# = 1$ for an open string dual, and $\# = 2$ for a closed string dual.

Even without SUSY, it is still an outstanding question whether $c$ can vary along conformal manifolds.
Beyond $a=c$

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1) In SCFTs, neither $a$ nor $c$ is a function of $\Delta_{\text{gap}}$. Instead, $a$-$c$ obeys a stronger bound

\[ \frac{|a - c|}{c} \lesssim \frac{1}{N\#} \]

where $\# = 1$ for an open string dual, and $\# = 2$ for a closed string dual.

Even without SUSY, it is still an outstanding question whether $c$ can vary along conformal manifolds.

2) It would also be nice to probe the couplings between the gravity and matter sectors. How are they, like $<TTT>$, constrained by the decoupled higher spin states?

[Anselmi, Freedman, Grisaru, Johansen]
Beyond $a=c$

There is another important property of the bulk:

$$S_{\text{bulk}} = \int (R + 2\Lambda) + \int (\partial_\mu \phi^i \partial^\mu \phi^i + \lambda_{ijk} \phi^i \phi^j \phi^k + \lambda_{ijkl} (\partial \phi^i \phi^j \phi^k \phi^l + \ldots)$$
Beyond $a=c$

There is another important property of the bulk:

$$S_{\text{bulk}} = \int (R + 2\Lambda) + \int \left( \partial_\mu \phi^i \partial^\mu \phi^i + \lambda_{ijk} \phi^i \phi^j \phi^k + \lambda_{ijkl} (\partial) \phi^i \phi^j \phi^k \phi^l + \ldots \right)$$

The gravity sector is not only universal, but isolated. That is, there exists a **consistent truncation** to the Einstein sector.

where $O$ is a light, single-trace operator not equal to $T$. 
Beyond $a=c$

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where $O$ is a light, single-trace operator not equal to $T$. 
The holographic dual of a derivative

• More robust than a-c: <TTO> can vary in 4d N<4 SCFT, even if O is protected.
The holographic dual of a derivative

• More robust than a-c: \( \langle T^{T \sigma} \rangle \) can vary in 4d N<4 SCFT, even if \( \mathcal{O} \) is protected.

\[ \text{Is } \langle T_{\mu \nu} T_{\rho \sigma} \mathcal{O} \rangle \sim \Delta_{\text{gap}}^{-2} ? \]

• This would furnish a CFT proof of consistent truncation to Einstein gravity, shown to follow from the absence of higher spin particles.

• In AdS, the first coupling that survives field redefinitions has four derivatives:

\[ S_{\text{bulk}} \supset \lambda_{TT\phi} \int \phi C_{\mu \nu \rho \sigma}^{2} \]
The holographic dual of a derivative

• More robust than a-c: $\langle TTO \rangle$ can vary in 4d N<4 SCFT, even if $O$ is protected.

$$\mathcal{I}_S \langle T_{\mu \nu} T_{\rho \sigma} O \rangle \sim \Delta^{-2}_{\text{gap}}?$$

• This would furnish a CFT proof of consistent truncation to Einstein gravity, shown to follow from the absence of higher spin particles.

• In AdS, the first coupling that survives field redefinitions has four derivatives:

$$S_{\text{bulk}} \supset \lambda_{TT} \phi \int \phi C^2_{\mu \nu \rho \sigma}$$

• In fact, we want to argue for a general avatar of “stringiness” in CFT. In string/M-theory, $\Delta_{\text{gap}}$ uplifts to 10/11d, where the low-energy limit yields a two-derivative action. This should be a generic consequence of large gap.

**Conjecture:** Counting AdS derivatives = Counting powers of $\Delta_{\text{gap}}$
Bounding TTO at large gap

Strategy: Impose unitarity on mixed systems of four-point functions of spinning operators in the Regge limit.

\[ \langle \Psi | \phi \phi | \Psi \rangle \]

- Conformal Regge theory computes the contribution of leading Regge trajectory, parameterized by \( j(\nu) \), to this CFT correlator.

- Take “mixed” state:

\[ |\Psi\rangle = |\epsilon \cdot T + c\mathcal{O}\mathcal{O}\rangle \]

- Unitarity requires the matrix of correlators to be positive, in a sense to be described.

- In the \( \Psi \Psi \rightarrow j(\nu) \rightarrow \phi \phi \) channel, positivity upper-bounds the off-diagonal couplings:

\[
\text{eig} \left( \begin{array}{cc} \langle T j(\nu) T \rangle & \langle T j(\nu) \mathcal{O} \rangle \\ \langle T j(\nu) \mathcal{O} \rangle & \langle \mathcal{O} j(\nu) \mathcal{O} \rangle \end{array} \right) \geq 0
\]

- \( T \) lives on this trajectory, at \( j=2 \). This bounds \( \langle \text{TTO} \rangle \).

- Imposing a large gap yields the desired behavior for \( \langle \text{TTO} \rangle \) and its dual AdS coupling.
Bounding TTO at large gap

$\psi (v) = \psi (-v)$

Stress tensor:

\[ j(-ih) = 2 \]

Definition of gap:

\[ j(-i(\Delta_{gap} - h)) = 4 \]

Large gap expansion:

\[ j(v) = 2 - \frac{v^2 + h^2}{\Delta^2_{gap}} + \ldots \]

\[ iv \equiv \Delta - h \text{, where } h \equiv \frac{d}{2} \]
Bounding TTO at large gap

Stress tensor:

\[ j(-i) = 2 \]

Definition of gap:

\[ j(-i(\Delta_{\text{gap}} - h)) = 4 \]

Large gap expansion:

\[ j(\nu) = 2 - \frac{\nu^2 + h^2}{\Delta_{\text{gap}}^2} + \ldots \]

We can consider OPE coefficients

\[ f(\nu) = \langle \mathcal{O}_1 \mathcal{O}_2 j(\nu) \rangle \]

A finite large gap limit implies

\[ f(\nu) \approx f(0) + \sum_{n=1}^{\infty} \frac{P_n(\nu^2)}{\Delta_{\text{gap}}^{2n}} , \text{ where } P_n(0) = 0 \]

\[ \rightarrow \text{Vanishing at the intercept implies } \Delta_{\text{gap}} \text{ suppression} \]

[Costa, Goncalves, Penedones; Caron-Huot; Costa, Hansen, Penedones; Kulaxizi, Parnachev, Zhiboedov; Li, Meltzer, Poland; Afkhami-Jeddi, Hartman, Kundu, Tajdini]
Bounding TTO at large gap

\[ \langle \epsilon \cdot T + c_\mathcal{O} \mathcal{O} | \phi \phi | \epsilon \cdot T + c_\mathcal{O} \mathcal{O} \rangle \]

- Take \( \mathcal{O} \) to be a scalar primary operator.
- Take the Regge limit. After the dust settles, unitarity leads to the following condition:

\[ \text{eig} \left( \mathcal{D}(\nu_0) \Pi_{\nu_0}(L) \right) \geq 0 \]

\( D \) is a matrix: \( i_j = T, \mathcal{O} \)

AdS\(_{d-1}\) propagator over geodesic distance \( L \)

In saddle-point approximation, the correlator is evaluated at:

\[ i\nu_0 \propto \frac{L}{\log S} \]

Sum over tensor structures

OPE coefficients

Regge limit of differential operators in spinning correlator
Bounding TTO at large gap

- \( \langle \text{TTO} \rangle \) – more generally, \( \langle \text{TTO} \text{(Spin j)} \rangle \) – has only one structure, and it has derivatives:

\[
\mathcal{D}(\nu_0) \Pi_{i\nu_0}(L) \approx \begin{pmatrix}
\beta_{T\Omega j(\nu)} & \beta_{T\Omega j(\nu_0)} \partial_L^2 \\
\beta_{T\Omega j(\nu)} \partial_L^2 & \beta_{\Omega\Omega j(\nu)}
\end{pmatrix} \Pi_{i\nu_0}(L)
\]

- Therefore, positivity requires off-diagonal suppression:

\[
\Pi_{i\nu_0}(L \ll 1) \sim L^{3-d}
\]

- Using our previous argument, we recover the desired result:

\[
\beta_{T\Omega j(\nu)} \approx c\nu^2 \Delta_{\text{gap}}^{-2} + \ldots \Rightarrow \beta_{T\text{TTO}} \sim \Delta_{\text{gap}}^{-2}
\]

- In this way, there is a direct correspondence between counting derivatives in CFT three-point structures, and derivatives in bulk effective actions.
To relate to AdS, need to show that $\beta$ is proportional to the AdS coupling

This identification has somewhat subtle dependence on $\Delta_\mathcal{O}$:

$$\beta_{T\mathcal{O}} = g(\Delta_\mathcal{O})C_{T\mathcal{O}} , \text{ where } g(2d + 2n) = 0$$

$$C_{T\mathcal{O}} = f^{-1}(\Delta_\mathcal{O})\lambda_{T\mathcal{O}} , \text{ where } f(2d + 2n) = 0$$

Zeroes of $f$ are nothing but the usual property of extremal correlators:

$$C_{T\mathcal{O}j(\nu)} = f_{\nu}^{-1}(\Delta_\mathcal{O})\lambda_{T\mathcal{O}j(\nu)} , \text{ where } f_{\nu}(\Delta_T + \Delta(\nu) + 2n) = 0$$

Therefore, the existence of a consistent truncation to Einstein gravity in a theory of gravity + scalar is a consequence of the absence of higher spin particles (for any scalar mass).
The following couplings are likewise suppressed by appropriate powers of $\Delta_{\text{gap}}$

<table>
<thead>
<tr>
<th>CFT</th>
<th>AdS</th>
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<tbody>
<tr>
<td>$\langle TTT \rangle_{R^2}$</td>
<td>$\int R_{\mu\nu\rho\sigma}^2$</td>
</tr>
<tr>
<td>$\langle TTT \rangle_{R^3}$</td>
<td>$\int R_{\mu\nu\rho\sigma}^3$</td>
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<tr>
<td>$\langle TT\phi \rangle$</td>
<td>$\int \phi R_{\mu\nu\rho\sigma}^2$</td>
</tr>
<tr>
<td>$\langle TJ\phi \rangle$</td>
<td>$\int R_{\mu\nu} D^\mu D^\nu \phi$</td>
</tr>
<tr>
<td>$\langle TT\phi \rangle_{\text{odd}} (3d)$</td>
<td>$\int \phi R_{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}^*$</td>
</tr>
<tr>
<td>$\langle TTJ \rangle_{\text{odd}} (4d)$</td>
<td>$\int A \wedge R \wedge R$</td>
</tr>
</tbody>
</table>

It would be satisfying to explicitly compute $\langle \text{TTO} \rangle$ in $1/\Delta_{\text{gap}}$ in specific examples = Derive cubic couplings in $\alpha'$-corrected AdS x M compactifications of string theory.

- e.g. Conifold CFT, dual to IIB on AdS$_5$ x T$_{1,1}$, has $\Delta=2$ (protected) and $\Delta=6$ (unprotected) scalars.

[Klebanov, Witten; Gubser; Ceresole, Dall’Agata, D’Auria, Ferrara]
Remarks

• Generically, in a theory of gravity coupled to matter, no truncation to Gauss-Bonnet gravity is allowed, not even at fixed order in low-energy perturbation theory: the GB and TTO couplings are set by the same scale.

  • (Genericity is required to ensure that a) TTO doesn’t vanish perturbatively in $\Delta_{\text{gap}}$, and that b) GB coupling is indeed controlled by $\Delta_{\text{gap}}$, not $1/N$.)
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• We have not studied $\langle\text{TTO}\rangle$ for $O$ in mixed symmetry representations of the Lorentz group. This would be required for a complete proof of sufficiency of large gap in CFT$_{d>3}$. However, we have proven this on the level of $\langle\text{TT}\varphi\varphi\rangle$ correlators.
TTO collider bounds

• The Regge method also allows derivation of conformal collider bounds: study the T point on the Regge trajectory, without imposing a large gap.

• For every correlator listed earlier, we get a universal upper bound.

• The bounds of TTO-type were recently derived by [Cordova, Maldacena, Turiaci] using the average null energy condition directly.
  • e.g. for scalar primary $O$

\[
\sum_{O} \frac{C_{TTO}^{2}}{C_{O}} f(\Delta) \leq N_{B} \quad \text{where} \quad f(\Delta) = \frac{(d-1)^3 d \pi^{2d} \Gamma \left( \frac{d}{2} \right) \Gamma (d+1) \Gamma (\Delta) \Gamma (\Delta - \frac{d-2}{2})}{(d-2)^2 \Gamma^4 \left( 2 + \frac{\Delta}{2} \right) \Gamma^2 \left( \frac{d+\Delta}{2} \right) \Gamma^2 \left( d - \frac{\Delta}{2} \right)}
\]

• (Double zeroes of $f$ explained via extremal correlator argument.)

• Extension to $O$ of spin $\geq 2$ underway. Potentially rich!
Concluding questions

**Take-home message:** CFT crossing and unitarity can reproduce AdS amplitudes & are responsible for features of AdS effective actions dual to large N, large gap CFTs.
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- Locality in $\text{AdS}_{d+1}$ vs $\text{AdS} \times M$: can we construct a fully explicit AdS/CFT dual pair with a local $\text{AdS}_{d+1}$ description?

- Find extra bulk dimensions from CFT.

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