BZ/CFT on BTZ

Electromagnetic energy extraction from rotating black holes and its holographic dual

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Collaboration with Maria Rodriguez, USU & MPI, Potsdam
(1709.10090)
Faraday disk – unipolar generator

\[ \Delta V = \int \mathbf{v} \times \mathbf{B} \cdot dl \]

\[ = \frac{1}{2} R^2 \Omega B \]

A reaction torque slows the disk.

(1831)
A pulsar is a spinning magnetized conductor

... with a $10^8$ Tesla field, and enormous electric field $E \sim R \Omega B \sim 10^{19} \text{ V/10 km}$

Generates an electron-positron plasma that forms a conducting circuit filled with “lamps”!

...and screens the electric field with a charge density $\sim \Omega B$

(Michel; Goldreich & Julian, 1969)
But how can there be a reaction force on a black hole?

Blandford-Znajek mechanism (1977)
Penrose process

“Gravitational collapse: the role of general relativity”
Roger Penrose (1969)
Mechanical analogy
(BZ, 1977)

Disk transfers angular momentum and energy to ring, and dissipates some energy as heat, increasing entropy. Black hole does the same to plasma, increasing horizon area.
Try to simplify BZ mechanism

- Cylindrical symmetry?
- 2+1 spacetime dimensions?
- No plasma?!
The simplest model

Next, surround the current by a rotating, cylindrical black hole...
Vacuum version of BZ mechanism

Spacetime metric:
\[ ds^2 = -\alpha^2 dt^2 + \alpha^{-2} dr^2 + r^2(d\phi - \Omega dt)^2 \]
\[ \alpha = \alpha(r), \quad \Omega = \Omega(r) \]

Electromagnetic field:
\[ * F = \frac{\dot{\Phi}}{2\pi \alpha^2 r} dr + \frac{Q}{2\pi} (d\phi - \Omega_F dt) \]
\[ F = -\frac{\dot{\Phi}}{2\pi} d\phi \wedge dt - \frac{Q}{2\pi r} dt \wedge dr - \frac{Q r (\Omega - \Omega_F)}{2\pi \alpha^2} (d\phi - \Omega dt) \wedge dr \]

Maxwell's equations:
\[ dF = 0, \quad d * F = 0 \]
Electric field

Magnetic field
Spacetime metric:
\[ ds^2 = -\alpha^2 dt^2 + \alpha^{-2} dr^2 + r^2 (d\phi - \Omega dt)^2 \]

Event horizon: \( \alpha(r_H) = 0 \)
\[ *F = \frac{\dot{\Phi}}{2\pi \alpha^2 r} \, dr + \frac{Q}{2\pi} (d\phi - \Omega_F dt) \]

Znajek horizon regularity condition: \( \dot{\Phi} = Qr_H(\Omega_H - \Omega_F) \)

**Azimuthal electric field determined by charge and magnetic field**

Energy extraction rate: \( Q^2 r_H \Omega_F (\Omega_H - \Omega_F) / 2\pi \)

*Direct parallel to the BZ mechanism...but with no plasma!*
Killing energy current:

\[ J_{\partial_t} = - (\partial_t \cdot F) \wedge *F + \frac{1}{2} \partial_t \cdot (F \wedge *F) \]

Flux through a constant \( r \) surface involves only the \( dr \) independent part:

\[ J_{\partial_t} \big|_r = - [(\partial_t \cdot F) \wedge *F]_r = \frac{\dot{\Phi} Q \Omega F}{4\pi^2} d\phi \wedge dt \]
Electric field on horizon-crossing timeslice of spinning BTZ black hole spacetime
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Energy extraction by cosmic string

Kinoshita, Igata, Tanabe 1610.08006
BZ/FT holography

Spinning BTZ black hole spacetime

\[ \alpha^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2 \ell^2}, \quad \Omega = \frac{r_- r_+}{r^2 \ell} \]

Black hole dual to a rotating thermal state at Hawking temperature
BZ/FT holography

Asymptotic vector potential

\[ A \xrightarrow{r \to \infty} \frac{\dot{\Phi}}{2\pi} t \, d\phi - \frac{Q}{2\pi} \ln r \left( dt - \ell^2 \Omega_F \, d\phi \right) \]

Corresponds to CFT current/source and gauge potential

\[ j^a = \frac{Q}{2\pi \ell} (\partial_t + \Omega_F \partial_\phi) \quad \text{and} \quad a = \frac{\dot{\Phi}}{2\pi} \, t \, d\phi. \]

Rotating charge density Electric field
BZ/FT holography

Conductivity is (current/electric field) in co-rotating frame

\[ \sigma = \frac{\hbar}{2\pi\gamma T_H} \]

\[ \gamma = (1 - \ell^2 \Omega_H^2)^{-1/2} \]

temperature in co-rotating frame

(Frolov & Thorne, 1989)

Joule heating of the CFT (\(\dot{Q} = (u \cdot j \cdot F)2\pi\ell\)) produces entropy increase dQ/T matching the horizon area/Bekenstein-Hawking entropy increase:

perfectly analogous to the BZ mechanical analogy!
Mechanical analogy
(BZ, 1977)

Disk transfers angular momentum and energy to ring, and dissipates some energy as heat, increasing entropy. Black hole does the same to plasma, increasing horizon area.
Electric field

Magnetic field

CFT thermal state rotates with $\Omega_H$

External charges rotate with $\Omega_F$
BZ/FT holography

Future directions?

• Instead of perturbing spinning neutral BTZ black hole, can perturb (known) spinning charged BTZ black hole in a magnetic field. The stationary solution co-rotates with the electric field, i.e. \( \Omega_H = \Omega_F \).

• Find time-dependent, (exact?) solution with \( \Omega_F \neq \Omega_H \) and azimuthal electric field. Study nonlinear conductivity (cf. Vaidya solutions of Horowitz, Iqbal, Santos, 1309.5088)

• FT interpretation of field singularity at inner horizon?

• BZ on black strings or branes in higher dimensions?

• Chern-Simons gauge field in 2+1?
In the ergosphere, the `time' translation symmetry becomes *spatial*. The corresponding globally conserved quantity is locally *momentum*, not energy, hence can be *negative*. Energy can be extracted by sending in negative energy: *Penrose Process*.

To remain stationary in the ergosphere, one must counter-rotate faster than light.
“Michel monopole” (1973)
spinning, conducting, magnetic monopole surrounded by force-free plasma

(from Beskin 2010)

B field lines
Electric field

Magnetic field

\[ E \times B \]