We need to talk (more) about $\text{AdS}_3 / \text{CFT}_2$

Based on: M. R. Gaberdiel, R. G. and C. Hull (1704.08665) and MRG+RG (in progress)
Siblings and their Issues

- Whole host of sibling AdS/CFTs: $AdS_5/CFT_4$, $AdS_4/CFT_3$, $AdS_2/CFT_1$.

- While sharing broad similarities, each has distinctive features.

- Each has had different development trajectories.

- Sometimes rapid progress on one front, sometimes another.

- And thus differing attention from the string community.

- All a part of life and growing up.
Three things:

• AdS$_3$/CFT$_2$ has been unduly languishing for a while and younger theorists are not so acquainted with her dual personalities.

• Somewhat overshadowed by her siblings.

• Despite having been in the limelight in the early years - success of Strominger-Vafa black hole microstate counting.

By AdS$_3$/CFT$_2$ I mean the collection of dualities involving superstring theories on AdS$_3 \times M$ dual to a 2d CFT.

• Will make a few observations on why it would be good to lavish a little more attention to this class of dualities.
Our understanding of AdS/CFT is very **incomplete** without a quantitative handle on **strings in AdS spacetime**.

A world sheet description appears to be **most tractable** for the case of $AdS_3$.

A complete description exists for the case of **NS-NS 3-form flux** (Maldacena-Ooguri + J. Son).

**Goal**: to achieve a **synthesis** with insights from the dual CFT, as well as integrability and hybrid worldsheet formalism.

**Focus on the spectrum**. Start with extreme regimes where the AdS is highly curved - i.e. strings are ```tensionless```. 
Basics of $\text{AdS}_3/\text{CFT}_2$

- The **maximally SUSY** backgrounds: focus mainly on $\text{AdS}_3 \times S^3 \times T^4$ but also $\text{AdS}_3 \times S^3 \times S^3 \times S^1$, $\text{AdS}_3 \times S^3 \times K3$.
- Arises from near horizon geometry of D1-D5 or F-String-NS5 system.
- In general, can have a **mixture** of NS-NS and RR 3-flux.
  \[
  \frac{R^2_{\text{AdS}}}{\alpha'} = \sqrt{Q^2_{NS} + g_s^2 Q^2_{RR}}
  \]
- At least in the case of pure RR flux, dual CFT believed to be on the moduli space of a **symmetric product orbifold** CFT.
- E.g. $(T^4)^N/S_N$, $(S_\kappa)^N/S_N$, $(K3)^N/S_N$, ...
Data Points

A) The spectrum at the symmetric orbifold point has a Cardy growth of massless HS fields: \( d(s) \propto e^{a\sqrt{s}} \).

Large unbroken symmetry (‘Higher Spin Square’). Spectrum (incl. twisted sectors) in representations of HSS.

B) Spectrum of NS-NS theory completely solved for any value of \( Q_{NS} = k \) (worldsheet \( SL(2, R)_k \) WZW theory).

Long Strings lead to a continuum of states - a special pathology due to pure NS flux. (Seiberg-Witten)

Nevertheless, valuable - see AdS physics from worldsheet.
C) Integrability: know mixed NS-NS/R-R spectrum in large spin limit (BMN).

\[ E - j = \sum_n N_n \sqrt{1 + \frac{2nQ_{NS}}{j} + \frac{n^2(Q_{NS}^2 + g_s^2Q_{RR}^2)}{j^2}} + \sqrt{Q_{NS}^2 + g_s^2Q_{RR}^2} N + \bar{N} \]

Spin chain description fixes the magnon like dispersion relation in pure RR case (like in higher dim.).

(David-Sahoo, OhlsonSax-Sfondrini, Stefanski, Torielli.....)

D) Hybrid worldsheet formalism for mixed flux (Green- Schwarz for \( AdS_3 \) and sphere, NS-R for rest).

(Berkovits-Vafa-Witten)

Supergroup principal chiral model (with WZW term).

Presence of a nonholomorphic current algebra may help.

(Ashok-Benichou-Troost)
Putting Together The Pieces

• Presence of two different fluxes leads to a more involved behaviour of the string spectrum as one varies parameters.

• At least two interesting “tensionless” limits: Symmetric orbifold point and a singular NS-NS limit ($Q_{NS} = k = 1$).

• Study possible relation between these two small radius limits.

• Would eventually like to piece together a global picture of the (non-BPS) spectrum as one moves away from these points.

• In contrast to higher dimensional cases: Straightforward interpolation of free gauge theory spectrum at $\lambda \ll 1$ to one where string states become very massive as $\lambda \gg 1$. 
A Novel Tensionless Limit

- If the symmetric product orbifold is a tensionless limit in the pure RR case, there seems to be a somewhat different tensionless limit in pure NS-NS case. (Gaberdiel-R.G.-Hull)

- The underlying physics can be seen even in the bosonic string based on an $SL(2, R)_k$ WZW model. (Maldacena-Ooguri)

- Spectrum has discrete representations $D_j^+ = \{|j, m\}$ of $SL(2, R)_k$ where $(\frac{1}{2} < j < \frac{k-1}{2})$ and there is a mass shell condition.

$$-\frac{j(j-1)}{k-2} + N_{sl(2)} + h_{int} = 1$$

- Spacetime energy and spin are $E = m + \bar{m}$; $s = |m - \bar{m}|$
A Novel Tensionless Limit (contd.)

• And continuous representations $\mathcal{C}(p, \alpha) = \{j = \frac{1}{2} + ip, m = \alpha + n\}$.

• As also spectrally flowed versions $\mathcal{D}^+_j; \mathcal{C}^+(w)(p, \alpha)$, labelled by an integer, based on generators: $\tilde{J}_n^\pm = J_n^\pm w$; $\tilde{J}_n^3 = J_n^3 + \frac{k}{2}w\delta_{n,0}$.

• The spectrally flowed discrete states give an infinite tower of discrete quasi primaries under the spacetime $\text{SL}(2,\mathbb{R})$.

• The spectrally flowed continuous states are long strings with winding number $w$ which have radial momentum $p$.

• Ask whether for some value of $k$, there are massless higher spin states. (i.e. obeying $E = \pm s$ with $s > 2$).
Massless higher spin states

• Thus \( E = -s \Rightarrow m = 0 \). One easily checks that the discrete representations (unflowed or flowed) cannot satisfy this.

• However, for the continuous representations, this condition implies satisfying a mass shell condition

\[
\frac{1}{4} + \frac{p^2}{k - 2} + \frac{k}{4}w^2 + N = 1
\]

• It is easy to verify that the only solution is for

\((p = 0, N = 0, w = 1, k = 3)\)

• Thus in the singly wound long string sector with \( k=3 \) and at the bottom of the continuum and no oscillator excitation.
Massless higher spin states (contd.)

- One then finds massless higher spin fields with spin \( s = \bar{N} \).
- In fact, the gravity multiplet is part of this family since precisely at \( k=3 \), it merges into the continuous representation.
- Like in the symmetric product case, there is a large oscillator tower of massless higher spin fields.
- Except that here they are each at the bottom of a continuum.
- Reflects the somewhat singular nature of this limit.
SUSY Cases

- The same phenomenon persists in the SUSY case (of $T^4$ and K3), except that now it occurs at $Q_{NS} = k = 1$. (Ferreira-Gaberdiel-Jottar)

- Appears somewhat strange since the effective level of the SU(2) theory is now $k_{eff} = -1$. Single NS5 brane theory.

- For the $AdS_3 \times S^3 \times S^3 \times S^1$ world sheet theory, $k=1$ limit corresponds to effective level of the SU(2) theories $= 0$.

- Note that for pure RR case, evidence for dual $(S_\kappa)^N/S_N$ CFT. (Eberhardt-Gaberdiel-Li)

- Thus for all the SUSY cases somewhat singular tensionless limit for highly curved AdS - can be seen from worldsheet.
Universality in the Tensionless Limit?

• Look more closely at this \( k=1 \) limit and make sense of it.

• Firstly, likely that some worldsheet d.o.f. are frozen.

• E.g. for \( AdS_3 \times S^3 \times S^3 \times S^1 \), bosonic levels of \( S^3 \)'s =0 implies that only excitations in \( AdS \) and \( S^1 \) directions.

• Physical state conditions remove two leave two bosonic d.o.f. (also eight fermionic d.o.f.).

• For \( T^4 \) case, \( k_{eff} = -1 \) implies \( c = -3 \) and again d.o.f. reduced.

• Plausible that oscillators of four bosons and four fermions. Try to compare to tensionless limit dual to symm. product.
Can we make a finer comparison of the spectrum at these two points? Note: dual CFT description vs worldsheet bulk.

Take the representative of the continuum at p=0 but for general winding \( w \). Mass shell cond:

\[
\frac{1}{4} + p^2 - w(m + \frac{w}{4}) + N = \frac{1}{2}
\]

Spacetime energy:

\[ h = m + \frac{1}{2}w = \frac{w}{4} + \frac{1}{w}(N - \frac{1}{4}) \]

Compare to the \( J \)-twisted sector of symm. product.

Oscillators with moding \( 1/J \). Identify \( w = J \).

Ground state energy: \( h_{gd} = \frac{w^2 - 1}{4w} \). Also matches with \( w = J \).

Finer matching after taking into account GSO projection.
Underlying Topological String?

- What does it mean to compare a part of the spectrum at the singular NS-NS point with the symmetric product?
- Perhaps a sense in which this subsector is consistent in itself.
- **Indication**: Consider the one loop thermal partition function on the worldsheet.
- Continuous representations arise from a set of discrete poles in the modular region.

\[ \tau = \frac{r}{w} + i \frac{\beta}{2\pi w} \] (Maldacena-Ooguri-Son)
Topological String (Contd.)

- Points correspond to **holomorphic maps** from worldsheet to boundary $T^2$.

- Now consider a worldsheet partition function which is a set of **delta function contributions** at these special values.

- This is **formally modular invariant** by itself.

- Does not have the continuum anymore.

- As in the **A-model Topological String** on $T^2$.

- Is the symm orbifold theory a topological string a la **BCOV**?

- Fits in with **free CFTs** being dual to a topological string.
To Summarise

- \(AdS_3\) is our **best bet** to understand the **stringy regime** of \(AdS\).
- Also has novel features like two **apparently different** tensionless limits - orbifold point vs. pure NS flux at \(k=1\).
- Moreover, there seem to be an interesting **correspondence** between the two limits.
- Is this due to the (same?) `Higher Spin Square` symmetry?
- Can we **move from the NS point** to the symmetric orbifold point preserving this symmetry?
- Can we describe the orbifold theory by a topological string?
A Big 25th Year Thank You to Princeton & My Academic Siblings

No Issues Here!
Thank You