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Introduction

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Harlow 1510.07911, Harlow/Ooguri 17xxxxx
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The arguments are actually quite simple, much of the work lies in defining carefully what we mean by gauge and global symmetries on the two sides. For the most part we will use language that applies equally well for continuous and discrete symmetries.
Global Symmetries

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1. Quantizing the theory on any spatial manifold $\Sigma$, there exists a unitary representation $U(g, \Sigma)$ of $G$ on the Hilbert space.
2. For any $O(x) \in L_x$, the set of local operators at $x \in \Sigma \times \mathbb{R}$, we have $U(g, \Sigma)^* O(x) U(g, \Sigma) \in L_x$.
3. For any $g \in G$ there is a local operator $O(x)$ on which $U(g, \Sigma)$ acts nontrivially.
4. For all $g \in G$ and for all $x \in \Sigma \times \mathbb{R}$, we have $U(g, \Sigma)^* T_{\mu\nu}(x) U(g, \Sigma) = T_{\mu\nu}(x)$. 


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When \( G \) is continuous and Noether’s theorem holds, these are obtained by exponentiating the integral of the current over \( R \). More generally, we will say that a symmetry which in addition to (1) – (4) also obeys (5) – (6) is splittable.
Not all symmetries are splittable in quantum field theory, but those which aren’t usually can be made so by adding degrees of freedom in the UV. Intuitively this follows from a simple theorem about finite tensor products:

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Let $U$ be a unitary operator on a Hilbert space $\mathcal{H} = \bigotimes_i \mathcal{H}_i$, with the property that it sends any operator which acts nontrivially only on one $\mathcal{H}_i$ to another such operator on the same $\mathcal{H}_i$. Then $U = \bigotimes_i U_i$. 
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We are thus free to take the product over only a subset of the $U_i$, producing the “lattice” version of $U(g, R)$. 

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In fact the existence of $U(g, R)$ can be proven formally within algebraic quantum field theory, provided one assumes the theory possesses a “splitting property” that is essentially a continuum version of the factorizability of the Hilbert space.

Buchholz/Doplicher/Longo
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- But on the other hand there are certain phenomena associated with gauge symmetries which are definitely physical. For continuous gauge groups in the Coulomb phase there are massless gauge bosons, while for discrete gauge groups it there can be a non-trivial topological field theory at low energies. I will call both of these “free charge phases”.

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- But on the other hand there are certain phenomena associated with gauge symmetries which are definitely physical. For continuous gauge groups in the Coulomb phase there are massless gauge bosons, while for discrete gauge groups it there can be a non-trivial topological field theory at low energies. I will call both of these “free charge phases”.
- If we have a gauge theory in the bulk which is not in its free charge phase, for example in a confining or Higgs phase, then it does not imply a global symmetry in the dual CFT.
Here is a definition which incorporates these comments:

Definition: A QFT on an infinite-volume spatial manifold $\Sigma$, with nontrivial asymptotic boundary $\partial \Sigma$, has gauge symmetry $G$ if:

- There exist a set of line operators $W^\alpha$, labeled by representations $\alpha$ of $G$, which we’ll call Wilson lines.
- There are boundary conditions at $\partial \Sigma$ such that for any spatial subregion $R$ of $\partial \Sigma$, there is a set of unitary operators $U(g, R)$ which act on the endpoints of any Wilson lines in $R$ with the appropriate representation matrices.

The theory allows finite energy charges, in the sense that the partition function on $S^1 \times \Sigma$ sees a finite ground-state energy shift if we wrap a Wilson line on the $S^1$. 
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Note that it might be possible for Wilson lines to end at local operators. If so then the theory will have charged states under $U(g, \partial \Sigma)$.
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A side comment: boundaries in QFT are important, just as we need to understand what happens when we put a QFT on a nontrivial compact manifold, we also need to understand what boundary conditions are possible.
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Claim: these are inconsistent.
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U(g, \Sigma) = U(g, R_1)U(g, R_2)U(g, R_3)U_{\text{edge}}.
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Since each $U(g, R_i)$ is localized in the boundary, it can only affect the bulk within the "entanglement wedge" of $R_i$. Since our charged operator is not in the entanglement wedge for any $R_i$, it must commute with all the $U(g, R_i)$ (and also $U_{\text{edge}}$). But then it must also commute with $U(g, \Sigma)$, which contradicts the assumption that the object is charged!
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This contradiction is easily avoided if we instead consider a *gauge* symmetry in the bulk: any charged operator then requires a Wilson line attaching it to the boundary, and this can be detected by the $U(g, R_i)$ for whichever $R_i$ the Wilson line ends in.
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Note that even if the object has finite size, by increasing the number of regions we can pull the entanglement wedges back to the boundary, so indeed this dressing really needs to make it all the way to infinity.
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- The algebra of these two is controlled by the algebra of the Wilson line and $U(g, R)$.
- This establishes that these $U(g, R)$ obey (1-6), up to showing that the charged operators transform in a faithful representation of the bulk gauge group (I’ll discuss this in a moment).
Conversely, if we assume that there is a global symmetry in the boundary theory then the \( U(g, R) \)'s give boundary conditions for a bulk gauge field, whose bulk equation of motion can then be solved (assuming a local semiclassical description with some effective action) to reconstruct the full set of surface operators in the bulk.
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- If $G$ is continuous then we can use $U(g, R)$ to extract the Noether current $J_\mu$, which is a rescaling of $F_{\mu\nu}$ at the boundary.
- For $\mathbb{Z}_p$ we can use the Banks-Seiberg Lagrangian

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to observe that

$$U(\theta, R) = e^{i\theta \int_R B},$$

which gives the boundary conditions for $B$ ($A$ is its canonical conjugate).
Conversely, if we assume that there is a global symmetry in the boundary theory then the $U(g, R)$'s give boundary conditions for a bulk gauge field, whose bulk equation of motion can then be solved (assuming a local semiclassical description with some effective action) to reconstruct the full set of surface operators in the bulk.

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which gives the boundary conditions for $B$ ($A$ is its canonical conjugate). (What about other discrete groups?)
Charged States

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**Theorem**

Let $G$ be a compact Lie group, and $\rho$ a faithful finite-dimensional representation of $G$. Then all irreducible representations of $G$ appear within the tensor product of some number of $\rho$’s and some number of $\rho^\ast$’s.
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**Theorem**

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Thus given an operator in faithful representation of $G$, we can act repeatedly with it (and its conjugate) on the vacuum until we have generated all the irreps (we can never get zero).
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What we need to rule out is the possibility that the symmetry group which acts faithfully on the CFT Hilbert space is really some quotient of the bulk gauge group by a discrete subgroup. For example could we have a bulk theory with gauge group $SU(2)$, but have the boundary theory only have symmetry group $SO(3) \cong SU(2)/Z_2$?
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Ruling this out will complete our above argument that a gauge theory in the bulk with gauge group $G$ implies a global symmetry group $G$ in the boundary theory.
Our first argument is based on one for $U(1)$ given in Harlow.
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For any $g$ there is some irreducible representation $\alpha$ for which $D_\alpha(g)$ is nontrivial, so we see that $U_R(g, \Sigma)$ must act nontrivially for all $g$, and thus act faithfully on the single-CFT Hilbert space.
Our second argument is less general, it works only when $G$ is connected, but it provides a useful alternative perspective.
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- This is related to the set of charged operators which exist in the CFT, since their correlation functions must be single-valued around any Dirac strings in the background gauge field.
- Since these background gauge fields are boundary conditions for the bulk gauge field, this relates the topology of the bulk gauge group to the set of charged local operators in just the right way to ensure that the bulk gauge group is represented faithfully on the CFT Hilbert space.
Compactness

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- There exist a finite set of primary operators $\mathcal{O}_i$ such that any other primary operator eventually appears in their iterated OPEs.
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This proposal is inspired by the idea that there should be finitely many “fundamental degrees of freedom”. This implies that there will some finite set of operators transforming in a faithful representation of $G$: I’ll call this representation $\rho: G \rightarrow U(N)$. 
Note that there ARE noncompact groups with finite-dimensional faithful unitary representations, for example $\mathbb{R}$ has the representation $(e^{ix}, e^{i\sqrt{2}x})$. The key point here is that this representation generates all the others.

The idea is to instead observe that the closure $\overline{G}$ in $U(N)$ of the image of $\rho$ is itself a Lie group, and since it is a closed subset of a compact space it is compact. Moreover by continuity the correlation functions of the operators in the representation $\rho$ will obey the selection rules for this larger symmetry group. In fact so will the correlation functions of the rest of the operators, since they are generated by products of these ones. So we can extend the representation $U(g, S_{d-1})$ to a representation of $\overline{G}$ on the full Hilbert space. Thus any putative noncompact symmetry group must really be a subgroup of a larger compact one!
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Conjecture: CFTs with discrete spectra and a unique stress tensor are always finitely generated.
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Conclusion

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Thanks!