Can CFT Compute the Minkowski Scattering Matrix?

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Tensor Network Renormalization and Holographic Spacetime
  TNRG for General CFT
  TNRG and HST

Black Holes and Fast Scrambling
  Minkowski to AdS transition from fast to Ballistic Scrambling
  TNRG Does Not Have the Transition From Fast to Ballistic Scrambling
Holographic Space-time

- Time Dependent Hamiltonian $H_{in}(t) + H_{out}(t)$ Acts on Hilbert Space of Nested Causal Diamonds (Proper Time Intervals) Along a Trajectory.
Causal Diamonds of a Geodesic in Anti-de Sitter Space
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- $H_{in}(t)$ Entangles More and More DOF of $H_{in}(\infty)$ as $t$ grows.
Tensor Network Renormalization

- Imagine One Has Ground State of Critical Lattice Model

- Iterate Two Steps: Unitary to Disentangle Short Range Correlations. Map Entangled Subsystem (Isometry on Full Space) to Coarse Grained Lattice.

- Evenbly-Vidal: Implement With Scale Dependent Hamiltonian, Choose Variationally Based on Explicit Fine Grained Lattice Model

- Numerical: Coarse Grained Hamiltonian Reproduces Low Lying $k_0 + P_0$ Spectrum of $H_{CFT}$
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FIG. 1: The tensor network structure of entanglement renormalization. Circles are lattice sites at various coarse grained scales. Squares with four lines are unitary disentaglers and triangles with three lines are isometric coarse graining transformations. The network shown here represents a $2 \rightarrow 1$ coarse graining scheme and has a characteristic fractal structure. In principle, each tensor can be different, but translation and scale invariance can provide strong constraints. This network implements a renormalization group transformation that is local in space and scale. This transformation has the important property that it coarse grains local operators into local operators.

Inspired by holography and the connection between entropy and geometry encoded in the ordinary boundary law, we will define a geometry from the entanglement structure of the quantum state. Imagine drawing boxes or cells around all the sites in the tensor network representing the quantum state as in Figure 2. We view these cells as units filling out a higher dimensional “bulk” geometry where the size of each cell is defined to be proportional to the entanglement entropy $S_{\text{site}}$ of the site in the cell. The connectivity of the geometry is determined by the wiring of the quantum circuit represented by the tensor network in Figure 2. The geometry ends whenever the coarse grained state completely factorizes. Now why is such a definition useful from the point of view of the full theory, and the quantum state is effectively extended into an emergent dimension representing scale. The network depends on $g$ because the ground state does.
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Look at Algebra of $G_i(t, \Omega)$ at fixed $t$. Keep only a finite number of terms in the harmonic expansion on the sphere. If there are "bosonic" $G_i$ with unbounded spectra for their Fourier modes, this must be cut off as well (c.f. scalar fields $\rightarrow$ spins in lattice models)
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- Field Theory RG wants to break rot inv.
- Diamonds much smaller than AdS radius see only low lying spectrum of $K_0 + P_0$. In models with large radius dual, spectrum evenly spaced.
Problems With $R_{AdS} \gg L_S$

- Black Hole Scrambling, As Seen From an Orbit $r - R_S \ll R_{AdS}$ Undergoes a Transition When $R_S \geq R_{AdS}$. Scrambling is fast below transition, then fast out to scales $R_{AdS}$ and ballistic on longer scales.
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- F.S. Requires Invariance Under (Fuzzy) Volume preserving maps.

- Another Problem: Low Energy Dimension Spectrum is Integrable so TNRG Hamiltonians Won’t Scramble At All.
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The $R_{AdS} \to \infty$ Limit

- Minkowski S Matrix Not Unitary in Fock Space

Due to the gapped spectrum in AdS, matrix elements with more than $R_{AdS}$ particles focused on the "arena" lead to black hole production.
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- For Shorter Times it is a Hamiltonian Invariant Under Fuzzy VPM and gets all the qualitative properties of Minkowski Black Holes right.
- An Explicit Model, Consistent With Unitarity, Locality, and Asymptotic AdS Symmetry (Wilson’s Argument about Emergent CFT), in which sub AdS radius scales are not probed by CFT correlators.
Conclusions

- TNRG Gives an Explicit Construction of HST Formalism for AdS/CFT for Bulk Holoscreens $\geq R_{AdS}$
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- DOF that scramble sub-AdS radius BHs in HST are those that become “0” energy gravitons in Minkowski space.
References

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