Quantum Focussing and the Quantum Null Energy Condition

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20 Years of AdS/CFT
Princeton, November 1, 2017
In the presence of black holes, the ordinary 2nd law becomes the Generalized Second Law:

\[ dS_{\text{gen}} \geq 0 , \]

where

\[ S_{\text{gen}} \equiv \frac{A}{4G\hbar} + S_{\text{out}} + \cdots ; \quad S_{\text{out}} = -\text{Tr}\rho_{\text{out}} \log \rho_{\text{out}} . \]

✓ Proof (semiclassical limit)

Wall 2011
GSL as Unification of Geometry & Information

The GSL is simultaneously a statement about geometry and about quantum info:

- It becomes Hawking’s area theorem in the classical limit.
- It becomes the ordinary second law in the case where there are no black holes.

But neither law survives on its own, if black holes are present and treated at the quantum level.
Stephen Hawking Discovers “2nd Law of Thermodynamics”

Claims It Follows From General Relativity
General Relativity as a Discovery Tool

1. Start with a classical gravity theorem involving area.
2. Add a quantum correction to make it robust against violations of the Null Energy Condition:
   \[ A \rightarrow A + 4G\hbar S_{\text{out}}. \]
3. Take a limit where gravity becomes unimportant.
4. Obtain a quantum law.

Can we actually do this, starting with other GR theorems?
General Relativity as a Discovery Tool

1. Start with a classical gravity theorem involving area.
2. Add a quantum correction to make it robust against violations of the Null Energy Condition:
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Can we actually do this, starting with other GR theorems? Yes: Classical Focussing Theorem \(\rightarrow\) Quantum Focussing
Generalized entropy can be defined not just for slices of event horizons... but for any 2D surface $\sigma$ that divides space into two sides.

This means we can consider what GR tells us about general surfaces. Let’s see what happens when we add a quantum correction, $A \rightarrow A + 4G\hbar S_{out}$ to appropriate GR formulas.
The classical expansion, $\theta$, is the (logarithmic) derivative of an area element, when transported along orthogonal light-rays.
Classical Focussing Theorem

In General Relativity, matter focusses light:

$$\theta' \leq 0.$$ 

Like the Area Theorem, this assumes the Null Energy Condition, $T_{kk} \geq 0$. Quantum effects can violate this. Example: evaporating black hole.

$\rightarrow$ Formulate a more robust, quantum-corrected focussing theorem!
Define a quantum expansion using \( A \rightarrow S_{\text{gen}} \equiv A + 4G\hbar S_{\text{out}} \):

\[
\Theta[\sigma; y_1] \text{ is the rate (per unit area) at which the generalized entropy changes when an infinitesimal area element of } \sigma \text{ at } y_1 \text{ is deformed in one of its future orthogonal null directions.}
\]

RB, Fisher, Leichenauer & Wall, 2015
Define a quantum expansion using $A \rightarrow S_{\text{gen}} \equiv A + 4G\hbar S_{\text{out}}$:

$\Theta[\sigma; y_1]$ is the rate (per unit area) at which the generalized entropy changes when an infinitesimal area element of $\sigma$ at $y_1$ is deformed in one of its future orthogonal null directions.  

RB, Fisher, Leichenauer & Wall, 2015
Define a quantum expansion using $A \rightarrow S_{\text{gen}} \equiv A + 4G\hbar S_{\text{out}}$:

$$
\Theta = \theta + \frac{4G\hbar}{A} S'_{\text{out}}.
$$

$\Theta[\sigma; y_1]$ is the rate (per unit area) at which the generalized entropy changes when an infinitesimal area element of $\sigma$ at $y_1$ is deformed in one of its future orthogonal null directions. 

RB, Fisher, Leichenauer & Wall, 2015
The classical expansion will not increase along any light-ray,

\[ \theta' \leq 0 , \]

if the NEC holds.
The quantum expansion will not increase along any light-ray,

$$\Theta' \leq 0,$$

regardless of whether the NEC holds.

RB, Fisher, Leichenauer & Wall, 2015
Quantum Focussing Conjecture

The QFC appears to be quite powerful. It implies:

1. Classical focussing theorem
2. Bekenstein’s GSL (and so Hawking’s area theorem) for black holes
3. Covariant Entropy Bound
4. New GSL for cosmology (and a new area theorem)
5. Quantum Null Energy Condition

I will briefly describe items 3 and 4, then 5 in more detail.
QFC Implies the Covariant Entropy Bound

Consider the case where the generalized entropy is initially decreasing away from the surface \( \sigma \).

Then the QFC implies that \( S_{\text{gen}} \) cannot increase anywhere along \( N \), and hence

\[ S_{\text{gen}}[\sigma'] \leq S_{\text{gen}}[\sigma]. \]
QFC Implies the Covariant Entropy Bound

Unpack $S_{\text{gen}} = S_{\text{out}} + A/4G\hbar \rightarrow$

$$\frac{A[\sigma] - A[\sigma']}{4G\hbar} \geq S_{\text{out}}[\sigma'] - S_{\text{out}}[\sigma].$$

For isolated matter systems on $N$, and in the special case where $A[\sigma'] = 0$, we recover the Covariant Entropy Bound, $S(N) \leq A/4G\hbar$. 
Area Theorem for Holographic Screens

A future holographic screen is a 2+1D hypersurface foliated by marginally trapped 2-surfaces $\sigma(r)$.
Area Theorem for Holographic Screens

Assume Einstein’s equations and the NEC. Then

\[ \frac{dA}{dr} > 0. \]

The area of a past or future holographic screen increases monotonically along its (unique) foliation.

✓ Proven, assuming the Null Energy Condition.

RB & Engelhardt, 2015a,b
2nd Law for Cosmology

Definition: A future (past) Q-screen is a hypersurface foliated by marginally quantum (anti-)trapped surfaces.

Conjecture: A past or future Q-Screen obeys the GSL:

\[ dS_{\text{gen}} \geq 0. \]

The cosmological 2nd law, too, is implied by the Quantum Focussing Conjecture.

RB & Engelhardt, 2015c
Entropy of a One-Sided Black Hole

Q: Holding fixed the exterior of the holographic screen, what maximizes the entropy of the entanglement patch?

Engelhardt & Wall, 2017
A: One can extend the light sheet inward until it includes a stationary surface of the same area.

Engelhardt & Wall, 2017
We lifted a GR theorem to a (semi-classical) quantum gravity conjecture,

$$\Theta' \leq 0.$$ 

Now, let’s throw away* the gravity part, and learn something new about QFT! We obtain the Quantum Null Energy Condition.
Expanding $\Theta$ into its classical and quantum part, we notice that the first term generally dominates, because it is $O(G^0)$.

For example, if the initial surface is a sphere in Minkowski space, $\theta = 2/R$. 

\[
\Theta = \theta + \frac{4G\hbar}{A} S'_{\text{out}}.
\]
We can suppress such geometric contributions to $\theta$, if we choose the initial surface to be a flat plane in Minkowski space. Then initially $\theta = 0$. 

From the QFC to the QNEC
From the QFC to the QNEC

$$\Theta = \theta + \frac{4G\hbar}{\mathcal{A}} S'_{\text{out}}.$$  

Away from the initial surface, $\theta$ will not vanish, because gravity bends the light-rays.

But now, the leading contributions to $\theta$ are $O(G)$, and so are of the same order as the “quantum correction.”
Finally, we can “tame” the effects of gravity by taking $G \to 0$. This ensures that only the $O(G)$ term contributes to $\theta$. It also means that $G$ cancels out as an overall factor. This is why the final result makes no reference to gravity at all. It is a QFT statement.
From the QFC to the QNEC

The QFC becomes

\[ 0 \geq \Theta' = \theta' + \frac{4G\hbar}{\mathcal{A}} (S''_{\text{out}} - S'_{\text{out}}\theta) \]

\[ = -\frac{1}{2} \theta^2 - \varsigma^2 - 8\pi G \langle T_{kk} \rangle + \frac{4G\hbar}{\mathcal{A}} (S''_{\text{out}} - S'_{\text{out}}\theta) \]

For a null surface with vanishing classical shear and expansion, \( \theta = \varsigma = 0 \), this implies

\[ \langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A} \to 0} \frac{S''_{\text{out}}}{\mathcal{A}} \quad (QNEC). \]
Quantum Null Energy Condition

\[ \langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A} \to 0} \frac{S''_{\text{out}}}{\mathcal{A}}. \]

First lower bound on the local energy density.

RHS: nonlocal, information-theoretic quantity.

Conversely, the local energy density limits how rapidly one can increase the rate at which information is acquired.
Quantum Null Energy Condition

\[ \langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} \lim_{A \to 0} \frac{S''_{\text{out}}}{A} \].

Since $G$ dropped out, we can try to prove this statement within QFT.
4.1 The Replica Trick

The replica trick prescription is to use the following formula for the von Neumann entropy [50]:

$$S_{\text{cut}} = - \text{Tr}[\rho \log \rho] - (1 - n \theta_{n}) \log \text{Tr}[\rho^{n}]$$ \quad \text{for} \quad n = 1, 2, \ldots \quad \text{(4.1)}$$

This can be written as

$$S_{\text{cut}} = \mathcal{D} \log Z_{n}$$ \quad \text{(4.2)}$$

where $Z_{n} \equiv \text{Tr}[\rho^{n}]$ and the operator $\mathcal{D}$ is defined by

$$\mathcal{D} f(n) \equiv (1 - n \theta_{n}) f(n)$$ \quad \text{for} \quad n = 1, 2, \ldots \quad \text{(4.3)}$$

where $f(n)$ is some function of $n$. Since $Z_{n}$ is only defined for integer values of $n$, we must analytically continue to real $n > 0$ in order to apply the $\mathcal{D}$ operator. The analytic continuation step is in general quite tricky, and will require care in our calculation. (Our analytic continuation is performed in Section 4.4.)

On general grounds discussed above, we must study the second-order term in a perturbative expansion of the entropy about the state $\rho^{(0)}$. Suppressing all $\lambda$ dependence, we have

$$Z_{n} = \text{Tr}[\rho^{(0)} + \sigma]^{n}$$ \quad \text{(4.4)}$$

Expanding $Z_{n}$ to quadratic order to isolate $\mathcal{O}^{(2)}$, we have

$$Z_{n} = \text{Tr}[\rho^{(0)}^{2}] + n \text{Tr}[\sigma(\rho^{(0)})^{2}] + \frac{n(n - 1)}{2} \text{Tr}[\sigma(\rho^{(0)})^{2}] + \cdots$$ \quad \text{(4.5)}$$

Using the notation introduced in (3.10) we can write

$$Z_{n} = \text{Tr}[\rho^{(0)}^{2}] + n \text{Tr}[\mathcal{O}(\rho^{(0)})^{2}] + \frac{n(n - 1)}{2} \text{Tr}[\mathcal{O}(\rho^{(0)})^{2}] + \cdots$$ \quad \text{(4.6)}$$

We denote by $\mathcal{O}^{(k)}$ the operator $\mathcal{O}$ conjugated by $(\rho^{(0)})^{k}$:

$$\mathcal{O}^{(k)} \equiv (\rho^{(0)})^{-k} \mathcal{O}(\rho^{(0)})^{k}$$ \quad \text{(4.7)}$$

$$d^{2} \mathcal{O}^{(k)} \equiv d^{2} \mathcal{O} \mathcal{O}^{(k)}$$ \quad \text{(4.8)}$$

In the replica trick case often works with the partition function $Z_{n}$ in terms of which $Z_{n} = Z_{n}(\mathcal{D})$. Choosing $Z_{n}$ over $\mathcal{O}$ is equivalent to choosing a different normalization for $\rho$, but we find it convenient to keep $\text{Tr} \rho = 1$.

Since $\mathcal{O}$ is the integral of operators with angles $0 < \theta < \pi$, it follows that $\mathcal{O}^{(k)}$ will be an integral over operators with angles $2\pi k < \theta < 2\pi (k + 1)$. Furthermore, since rotations by $2\pi k$ commute with translations by $\lambda$, we can obtain $\mathcal{O}^{(k)}$ from $\mathcal{O}$ simply by letting the range of integration that defines $\mathcal{O}$ shift from $[0, 2\pi]$ to $[2\pi k, 2\pi (k + 1)]$, as long as we define $f_{n}(r, \theta)$ to be periodic in $\theta$ with period $2\pi r$.

It will also be convenient to introduce an angle-ordered expectation value, defined as

$$\langle \cdots \rangle_{\theta} \equiv \frac{\text{Tr}[\rho^{(0)}^{n} T[\cdots]]}{\text{Tr}[\rho^{(0)}^{n}]}$$ \quad \text{(4.9)}$$

where $T[\cdots]$ is $\theta$-ordering. Then (4.6) can be written

$$Z_{n} = \text{Tr}[\rho^{(0)}^{n}] \left( 1 + n \langle \mathcal{O} \rangle_{\theta} + \frac{n(n - 1)}{2} \langle \mathcal{O}^{2} \rangle_{\theta} \right) + \cdots$$ \quad \text{(4.10)}$$

Taking the logarithm of $Z_{n}$ and extracting the part quadratic in $\sigma$ gives

$$\log Z_{n} \geq \frac{n(n - 1)}{2} \langle \mathcal{O}^{2} \rangle_{\theta} - \frac{n^{2}}{2} \langle \mathcal{O} \rangle_{\theta}^{2}$$ \quad \text{(4.11)}$$

where we have kept only the part quadratic in $\mathcal{O}$. The contribution of the second term to the entanglement entropy will be proportional to $\langle \mathcal{O} \rangle_{\theta}$, which vanishes because of the tracelessness of $\sigma$. Therefore we only need to consider the first term.

Since we are considering angle-ordered expectation values, we have the identity

$$\langle \left( \sum_{k=0}^{n-1} \mathcal{O}^{(k)} \right)^{2} \rangle_{\theta} = n \langle \sum_{k=0}^{n-1} \mathcal{O}^{(k)} \mathcal{O}^{(k)} \rangle_{\theta}$$ \quad \text{(4.12)}$$

and so from the first term in (4.11) the relevant part of $\log Z_{n}$ can be written as

$$\log Z_{n} \geq -\frac{n}{2} \langle \mathcal{O} \mathcal{O} \rangle_{\theta} + \frac{n - 1}{2} \left( \sum_{k=0}^{n-1} \langle \mathcal{O}^{(k)} \mathcal{O}^{(k)} \rangle_{\theta} \right)$$ \quad \text{(4.13)}$$

Restoring the $\lambda$ dependence and taking $\lambda$ derivatives gives

$$S^{\theta}_{\text{cut}} = \frac{\partial^{2}}{\partial \lambda^{2}} \left| \mathcal{D} \log Z_{n}(\lambda) \right.$$ \quad \text{(4.14)}$$

$$S^{\theta}_{\text{cut}} = -\frac{n}{2} \langle \mathcal{O} \mathcal{O} \rangle_{\theta} + \frac{1}{2} \left( \sum_{k=0}^{n-1} \langle \mathcal{O}^{(k)} \mathcal{O}^{(k)} \rangle_{\theta} \right)$$ \quad \text{(4.15)}$$

One could worry that the phase factor in (3.5) spoils this relation, but notice that the phase has period $2\pi$ in $\theta$ and do not appear when shifting by $2\pi k$.\footnote{One could worry that the phase factor in (3.5) spoils this relation, but notice that the phase has period $2\pi$ in $\theta$ and do not appear when shifting by $2\pi k$.}
Proof for Free Fields

Using (3.11), we can write the sum over replicas in (4.24) as follows:

\[ \left\langle \sum_{n=0}^{n-1} a^{(n)} \right\rangle_n = \left\langle \sum_{n=0}^{2n\pi} dr \, dB \, f_{ij}(r, \theta) \partial \Phi(r, \theta, \lambda) \otimes E_{ij}(\theta) \right\rangle_n. \]  

(4.25)

This equality comes from interpreting \( a^{(n)} \) as \( a \) inserted on the \((k+1)\)th replica sheet (see (4.7)). Summing over sheets and integrating \( \theta \in [0, 2\pi] \) on each one is equivalent to just integrating \( \theta \in [0, 2\pi n] \), which covers the entire replicated manifold. The definition of \( \partial \Phi \) for angles greater than \( 2\pi \) is given by the Heisenberg evolution rule, the right-hand side of (3.5). The field is still holomorphic, but it would be misleading to write it as a function of \( \theta \) since it is not periodic in \( \theta \) with period \( 2\pi \).

Because the \( f_{ij}(r, \theta) \) are not dynamical, they should be identical on each sheet. In the Fourier representation as in (4.21), this means keeping the Fourier coefficients fixed and keeping the \( m \) parameters integral. Thus we have

\[ \frac{1}{Z \Phi} \left( \sum_{n=0}^{n-1} a^{(n)} \right)_n = \frac{1}{Z \Phi} \sum_{m \in \mathbb{Z}} \left( \int \frac{dr \, dB \, f_{ij}^{\text{ren}}(r) f_{ij}^{\text{ren}}(r') e^{-im\theta - im\phi} \partial \Phi(r, \theta, \lambda)}{(2\pi)^2} \right) \left( E_{ij}(\theta) E_{ij}(\theta') \right)_n. \]  

(4.26)

The CFT two point function is calculated in Appendix A.1:

\[ \langle \partial \Phi(z) \partial \Phi(w) \rangle_n = \frac{1}{n(zw)^2} \sum_{q \in \mathbb{Z}} \text{sign}(q) q^2 (q^2 - 1) \left( \frac{z}{w} \right)^q. \]  

(4.27)

where \( q \) takes values in the integers divided by \( n \), and

\[ P(q, r, r') = \text{sign}(q) q^{r - r'}. \]  

(4.28)

When \( n = 1 \) there are no nonzero terms in the sum, but when \( n > 1 \) the answer is nonzero. For future convenience, we separated the parts which depend on \( \theta \) from those that do not.

The auxiliary system two point function is calculated in Appendix A.2:

\[ \langle E_{ij}(\theta) E_{ij}(\theta') \rangle_n = \delta_{ij} \delta_{r r'} e^{-2 \pi \xi n^2 K_n} \sum_p e^{\phi(p, q - q') \sin \theta n \alpha, \alpha + \alpha_j} e^{\phi(p - \alpha, q + \alpha_j)}, \]  

(4.30)

where \( p \) is also an integer divided by \( n \) and \( \frac{2 \pi n K_n}{\alpha} \) is a normalization factor. Substituting this equation as well as (4.28) into (4.26) gives

\[ \mathcal{D} = \frac{1}{n^2(2\pi)^2 Z_n^2} \sum_{m \in \mathbb{Z}} \left( \int \frac{dr \, dB \, f_{ij}^{\text{ren}}(r) f_{ij}^{\text{ren}}(r') e^{-im\theta - im\phi} \partial \Phi(r, \theta, \lambda)}{(2\pi)^2} \right) \left( E_{ij}(\theta) E_{ij}(\theta') \right)_n. \]  

(4.31)

The angle integrations give Kronecker deltas multiplied by \( 2\pi \). The result is

\[ \mathcal{D} = \frac{-i}{2\pi \alpha} \sum_{m \in \mathbb{Z}} \left( \int \frac{dr \, dB \, f_{ij}^{\text{ren}}(r) f_{ij}^{\text{ren}}(r') e^{-im\theta - im\phi} \partial \Phi(r, \theta, \lambda)}{(2\pi)^2} \right) \left( E_{ij}(\theta) E_{ij}(\theta') \right) \]  

(4.32)

In going to the last line, we used the fact that the sum in brackets vanishes when \( n = 1 \) and that, for any two functions \( f(u), g(u) \) such that \( f(1) \) and \( \left[ \frac{d}{du} f(u) \right]_{u=1} \) are finite and \( g(1) = 0 \), the following relation holds:

\[ \mathcal{D}(f(g)(n)) = f(1) \mathcal{D}_{g}(n). \]  

(4.33)

We now turn to the analytic continuation and application of \( \mathcal{D} \) on the term in brackets in (4.32). We will take care of the awkward \( \text{sign}(q) \) by writing the \( q \)-dependent part of the sum as two sums with positive argument. We will suppress the \((r, r')\) dependence for the rest of the calculation:

\[ \sum_{m \in \mathbb{Z}} \text{sign}(q) P(q) = \sum_{m \in \mathbb{Z}} \frac{P(q) + P(-q)}{q + m + i\alpha_j} \]  

(4.34)

Now we write \( q = k/n \) to turn this into a sum over integers:

\[ \sum_{k \in \mathbb{Z}} \left( \frac{P(q)}{q + m + i\alpha_j} + \frac{P(-q)}{q - m - i\alpha_j} \right) \]  

(4.35)

In the next section we will see how to evaluate and analytically continue such sums quite generally.
Proof for Free Fields

- applies to free or superrenormalizable bosonic fields, stationary null surfaces
- null quantization $\rightarrow$ operator algebra factorizes over generators ("pencils")
- each pencil is 1+1 CFT
- in any global finite energy state, individual pencils are near the vacuum $\rightarrow$ small expansion parameter

RB, Fisher, Koeller, Leichenauer & Wall, 2015
Proof for Free Fields

- expand state, $\rho = \rho_0 + \sigma(\lambda)$
- expand entropy in powers of $\sigma$, $S = \sum S^{(i)}$
- find that $(S^{(0)} + S^{(1)})''$ would saturate the QNEC
- compute $S^{(2)''}$ using replica trick, prove $< 0$.

RB, Fisher, Koeller, Leichenauer & Wall, 2015
Interacting theories with a gravity dual satisfy the QNEC. Koeller & Leichenauer, 2015

This follows
- from entanglement wedge nesting (Wall 2012),
- which in turn is necessary for consistent subregion duality.
Extension to Higher Curvature Gravity

Explored by Fu, Koeller, Marolf (2017a)

Extension to Curved Space

Explored by Fu, Koeller, Marolf (2017b).

Soon after, resolved by Akers, Chandrasekharan, Leichenauer, Levine, Shahbazi Moghaddam (2017): the same conditions on the geometry are necessary and sufficient for all three of the following statements:

- $\text{QFC} \implies \text{QNEC}$
- $\text{AdS/CFT} + \text{RT} \implies \text{QNEC}$ for theories with gravity dual
- $\text{QNEC}$ well-defined
A General Proof of the Quantum Null Energy Condition

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Abstract

We prove a conjectured lower bound on $\langle T_+(x) \rangle_\psi$ in any state $\psi$ of a relativistic QFT dubbed the Quantum Null Energy Condition (QNEC). The bound is given by the second order shape deformation, in the null direction, of the geometric entanglement entropy of an entangling cut passing through $x$. Our proof involves a combination of the two independent methods that were used recently to prove the weaker Averaged Null Energy Condition (ANEC). In particular the properties of modular Hamiltonians under shape deformations for the state $\psi$ play an important role, as do causality considerations. We study the two point function of a “probe” operator $O$ in the state $\psi$ and use a lightcone limit to evaluate this correlator. Instead of causality in time we consider causality in modular time for the modular evolved probe operators, which we constrain using Tomita-Takesaki theory as well as certain generalizations pertaining to the theory of modular inclusions. The QNEC follows from very similar considerations to the derivation of the chaos bound and the causality sum rule. We use a kind of defect Operator Product Expansion to apply the replica trick to these modular flow computations, and the displacement operator plays an important role. Our approach was inspired by the AdS/CFT proof of the QNEC which follows from properties of the Ryu-Takayanagi (RT) surface near the boundary of AdS, combined with the requirement of entanglement wedge nesting. Our methods were, as such, designed as a precise probe of the RT surface close to the boundary of a putative gravitational/stringy dual of any QFT with an interacting UV fixed point. We also prove a higher spin version of the QNEC.
Proof for Interacting Fields

Combines techniques used in two recent proofs of the ANEC (from quantum info/from causality):

- Modular flow
- Monotonicity of the full modular Hamiltonian
Ongoing Work

Saturation of the Diagonal QNEC?

Holographic computations

Leichenauer, Levine, Shahbazi Moghaddam
(See also Ecker, Grumiller, van der Schee, Stanzer 2017)
Ongoing Work

Saturation of the Diagonal QNEC?

Holographic computations

Leichenauer, Levine, Shahbazi Moghaddam

(See also Ecker, Grumiller, van der Schee, Stanzer 2017)

Defect OPE

+Faulkner, Chandrasekharan, Balakrishnan