CENTRAL CHARGE AND CORRELATORS IN STRINGS ON ADS3 WZW

- LIGHTNING REVIEW OF THE ADS3 WZW
- STRINGS ON ADS3 WZW: HOLOGRAPHY BEYOND THE SUPERGRAVITY LIMIT
- THE SPACETIME AFFINE LIE AND VIRASORO ALGEBRAS
- THE IDENTITY OPERATOR
- THE HOLOGRAPHIC DICTIONARY
- A PROBLEM WITH CLUSTER DECOMPOSITION AND WARD IDENTITIES
- THE SOLUTION: “THE CONUNDRUM IS A BUSILLIS”
- CLUSTER DECOMPOSITION RECOVERED
- A FEW FINAL OBSERVATIONS
LIGHTNING REVIEW OF THE ADS3 WZW
(SEE GIVEON, KUTASOV, SEIBERG)

ADS3 CLASSICAL METRIC

\[ ds^2 = k(d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma}) \]

CLASSICAL LAGRANGIAN

\[ L = ke^{2\phi}(\partial\phi\bar{\partial}\phi + \bar{\partial}\gamma\partial\bar{\gamma}) \]
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AN ANTISYMMETRIC B FIELD HAS BEEN ADDED (WZ TERM)

SYMMETRY OF THEORY IS SL(2,R) \times SL(2,R)
CLASSICAL PRIMARY FIELDS BELONGING TO REPRESENTATION $j=h-1$ of SL(2,R) [LEFT-RIGHT SYMMETRIC]

ARE SOLUTIONS OF THE TARGET-SPACE LAPLACE EQUATION DESCRIBING SCALARS PROPAGATING ON THE BACKGROUND

THEY ARE TARGET-SPACE FIELDS OF DIMENSION $(h,h)$

$$\Phi_h = (|\gamma - x|^2 e^\phi + e^{-\phi})^{-2h} \approx \frac{1}{2h-1} e^{2(h-1)\phi} \delta^2(\gamma - x) + \mathcal{O}\left(e^{2(\phi-2)}\right)$$
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HOLOGRAPHICALLY: SOURCES FOR OPERATORS AT BOUNDARY POINT $x$
IN THE FULL QUANTUM THEORY $\Phi_h$ BECOME
WORLDSHEET FIELDS LABELED BY $h,x$

THE STRING THEORY VERTICES ARE

$$V(x, \bar{x}, h, I) = \int d^2z \Phi_h(x, \bar{x}|z, \bar{z}) O_I \quad \Delta_I + \Delta_h = 1$$
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INSTEAD OF IN AND OUT STATES WE HAVE “LOCAL OPERATORS” LABELED BY BOUNDARY COORDINATES
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WORLDSHEET CURRENT ALGEBRA COMPactly WRITTEN AS

$$J(x|z) = 2xJ_3(z) - J^+(z) - x^2 J^-(z)$$
SPACETIME VIRASORO AND AFFINE-LIE ALGEBRAS ARE GENERATED BY STRING VERTICES THAT GIVE RISE TO THE CORRECT WARD IDENTITIES

AFFINE-LIE (SIMPLER TO WRITE)

THEY ARE PRESENT WHENEVER THE COMPACTIFICATION IS

\[ AdS_3 \times S \]
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SYMMETRIC SPACE WITH WORLD-SHEET CURRENTS

\[ k^a(z), k^a(\bar{z}) \]
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AFFINE-LIE (SIMPLER TO WRITE)

\[ K^a(x) = -\frac{1}{k} \int d^2 z k^a(z) \bar{J}(\bar{x}|\bar{z}) \Phi_1(x, \bar{x}|z, \bar{z}) = -\frac{1}{\pi} \int d^2 z \bar{\partial}[k^a(z) \Lambda(x, \bar{x}|z, \bar{z})] \]
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A BAD OPERATOR WITH log(z) CORRELATOR
WARD IDENTITY IN VEVS OF VERTICES

$$\langle \ldots K^a(x) K^b(y) \ldots \rangle = \langle \ldots \frac{1}{(x - y)^2} k_G I + \frac{1}{(x - y)} f^{ab}_c K^c(y) \ldots \rangle$$

IDENTITY OPERATOR:

$$I = \frac{1}{k^2} \int d^2 z J \bar{J} \Phi_1 = -\frac{1}{2\pi i k} \oint dz J \Lambda$$
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INDEPENDENT OF \( x, \bar{x} \) BUT NOT SAME CONSTANT ON DIFFERENT VEVS OF PHYSICAL VERTICES

\[ \langle I \prod_i \Phi_{h_i} \rangle = \frac{1}{k} [1 - g + \sum_i (h_i - 1)] \langle \prod_i \Phi_{h_i} \rangle \]
WARD IDENTITY IN VEVS OF VERTICES

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g=0 GIVEON-KUTASOV, ALL g, KIM-PORRATI
\[ W = \sum_{g=0}^{\infty} g_s^{2g-2} \langle \exp \left[ \int d^2 x J(x, \bar{x}, h, I) V(x, \bar{x}, h, I) + \int d^2 x \lambda(x, \bar{x}) I \right] \rangle_g \]
THE HOLOGRAPHIC DICTIONARY (de Boer, H. Ooguri, H. Robins, J. Tannenhauser)

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GENERATING FUNCTIONAL FOR CONNECTED BOUNDARY CORRELATORS
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GENERATING FUNCTIONAL FOR CONNECTED BOUNDARY CORRELATORS

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GENERATING FUNCTIONAL FOR BOUNDARY CORRELATORS

SO THE IDENTITY OPERATOR HAS NONZERO CONNECTED CORRELATORS E.G.

\[ \langle I \Phi_h \Phi_h \rangle_{g=0} = \frac{1}{k} (2h - 1) \langle \Phi_h \Phi_h \rangle_{g=0} \]
ADD SOURCES FOR VIRASORO AND AFFINE-LIE ALGEBRAS

\[ W = \sum_{g=0}^{\infty} g_s^{2g-2} \langle \exp \left[ \int d^2 x \left( J(x, \bar{x}, h, I) V(x, \bar{x}, h, I) + K_a(x) A_{\bar{x}}^a(x, \bar{x}) + T_{xx}(x) g_{\bar{\bar{x}}} (x, \bar{x}) + \lambda(x, \bar{x}) I \right) \right] \rangle_g \]
ADD SOURCES FOR VIRASORO AND AFFINE-LIE ALGEBRAS

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\]
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\[ W = \sum_{g=0}^{\infty} g_S^{2g-2} \exp \left[ \int d^2 x \,(J(x, \bar{x}, h, I)V(x, \bar{x}, h, I) + K_{ax}(x)A_x^a(x, \bar{x}) + T_{xx}(x)g_{\bar{x}\bar{x}}(x, \bar{x}) + \lambda(x, \bar{x})I) \right] \langle g \right] \]

WARD IDENTITIES RECOVERED BY TRANSFORMING SOURCES AS

\[ \delta_\epsilon J = T J, \quad \delta_\epsilon A_{\bar{x}} = D_{\bar{x}}^a \epsilon_a, \quad \delta_\epsilon \lambda = -\pi k_G \epsilon_a \partial_x A^q_{\bar{x}} \]
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\( \lambda \) IS A GREEN-SCHWARZ FIELD
WARD IDENTITY

\[ \delta_\epsilon J \ast \frac{\delta W}{\delta J} + \delta_\epsilon A \ast \frac{\delta W}{\delta A} = - \int d^2 x \delta_\epsilon \lambda \frac{dW}{d\lambda_0} \]
WARD IDENTITY

\[ \delta_\epsilon J \star \frac{\delta W}{\delta J} + \delta_\epsilon A \star \frac{\delta W}{\delta A} = - \int d^2 x \delta_\epsilon \lambda \frac{dW}{d\lambda_0} \]

PROBLEM: THE CORRECT WARD IDENTITY IS

\[ \delta_\epsilon J \star \frac{\delta W}{\delta J} + \delta_\epsilon A \star \frac{\delta W}{\delta A} = \text{constant} \times \int d^2 x \delta_\epsilon \lambda \]
PROBLEM: CLUSTER FACTORIZATION IS LOST

\[ \langle \Phi_{h_1}(x_1) \Phi_{h_2}(x_2) \Phi_{h_3}(x_3) \Phi_{h_4}(x_4) \rangle \rightarrow \langle \Phi_{h_1}(x_1) \Phi_{h_2}(x_2) I \rangle \frac{1}{\langle II \rangle} \langle I \Phi_{h_3}(x_3) \Phi_{h_4}(x_4) \rangle + \ldots \]
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\]

MORE GENERALLY (MALDACENA-OOGURI):

WHEN

\[
h_i + h_j < (k + 1)/2
\]

4-POINT FUNCTION FACTORIZES ON ALL OPERATORS WITH

\[
1/2 < h < (k + 1)/2
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4-POINT FUNCTION FACTORIZES ON ALL OPERATORS

WITH

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I HAS \( h = 1 \) AND IS INDEPENDENT OF \( x \)
“THE CONUNDRUM IS A BUSILLIS”
AN ENTRY IN THE HOLOGRAPHIC DICTIONARY MUST BE CORRECTED
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AN ENTRY IN THE HOLOGRAPHIC DICTIONARY MUST BE CORRECTED

$$\langle I \rangle \sim \text{number of F-strings creating the } \text{AdS}_3 \text{ background } \sim c$$

IT MUST BE KEPT FIXED, WHILE THE “FREE ENERGY” $W$ KEEPS THE CHEMICAL POTENTIAL $\lambda$ FIXED
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IT MUST BE KEPT FIXED, WHILE THE "FREE ENERGY" \( W \) KEEPS THE CHEMICAL POTENTIAL \( \lambda \) FIXED

SOLUTION: DEFINE GENERATOR OF CONNECTED CORRELATORS BY LEGENDRE-TRANSFORMING IN \( \lambda \)

\[ \Gamma[\langle I \rangle, J] = W[\lambda_0, J] - \lambda_0 \langle I \rangle, \text{ computed at } \frac{dW}{d\lambda_0} = \langle I \rangle \]
PROPERTIES OF THE “EFFECTIVE ACTION”
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\[ \delta_\epsilon J * \frac{\delta \Gamma}{\delta J} + \delta_\epsilon A * \frac{\delta \Gamma}{\delta A} = -\langle I \rangle \int d^2x \delta_\epsilon \lambda \]
PROPERTIES OF THE “EFFECTIVE ACTION”

\[ \delta_\epsilon J * \frac{\delta \Gamma}{\delta J} + \delta_\epsilon A * \frac{\delta \Gamma}{\delta A} = -\langle I \rangle \int d^2x \delta_\epsilon \lambda \]

CORRECT VALUE OF THE CENTRAL CHARGE
PROPERTIES OF THE “EFFECTIVE ACTION”

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CORRECT VALUE OF THE CENTRAL CHARGE

\[ \Gamma_{1234} = W_{1234} - W_{12\lambda} \frac{1}{W_{\lambda\lambda}} W_{\lambda34} + \ldots \]

CLUSTER DECOMPOSITION VIOLATING TERMS (INTERNAL \( \lambda \) LINES) CANCEL
PROPERTIES OF THE “EFFECTIVE ACTION”

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CLUSTER DECOMPOSITION VIOLATING TERMS (INTERNAL \(\lambda\) LINES) CANCEL

THIS PROPERTY HOLDS IN GENERAL BECAUSE THE EFFECTIVE ACTION IS 1-PI IN \(\lambda\)
A FEW FINAL COMMENTS

THE CORRECT HOLOGRAPHIC PRESCRIPTION IS

\[ Z = C e^{\Gamma[\langle I \rangle, J]} \]
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WE DEFINED OUR WORLD-SHEET THEORY PERTURBATIVELY AROUND THE POINT

\[ J = 0, \lambda = 0 \]

THIS IS THE POINT WHERE

\[ \langle I \rangle = \frac{dW}{d\lambda_0} \bigg|_{J=\lambda_0=0} = \frac{1}{k} \]
A FEW FINAL COMMENTS

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THE POINT IS DEFINED BY THE VANISHING-TADPOLE CONDITION:

\[ \left. \frac{d\Gamma[\Psi, J]}{d\Psi} \right|_{J=0} = 0 \]
OUR PRESCRIPTION CLOSELY RESEMBLES DEFINING
LIOUVILLE THEORY AT FIXED AREA (IN WAKIMOTO
REPRESENTATION I IS SIMILAR TO THE AREA OPERATOR)
OUR PRESCRIPTION CLOSELY RESEMBLES DEFINING LIOUVILLE THEORY AT FIXED AREA (IN WAKIMOTO REPRESENTATION \( I \) IS SIMILAR TO THE AREA OPERATOR)

WHEN COMPUTING ENTROPIES OF BPS BLACK HOLE MICROSTATES BEYOND THE LEADING APPROXIMATION, SIMILAR ISSUES ARISE. IN THAT CASE THEY CENTER ON THE CORRECT DEFINITION OF THE SETS OF CHARGES AND CHEMICAL POTENTIALS TO KEEP FIXED
WHEN COMPUTING ENTROPIES OF BPS BLACK HOLE MICROSTATES BEYOND THE LEADING APPROXIMATION, SIMILAR ISSUES ARISE. IN THAT CASE THEY CENTER ON THE CORRECT DEFINITION OF THE SETS OF CHARGES AND CHEMICAL POTENTIALS TO KEEP FIXED.

WE WORKED IN THE $k > 1$ CASE. FOR $k < 1$ THE IDENTITY IS NOT A PHYSICAL OPERATOR AND DOES NOT HAVE 3-POINT FUNCTIONS WITH PHYSICAL VERTICES. SO, IT IS NOT CLEAR IF ONE MUST LEGENDRE-TRANSFORM EVEN IN THE LATTER CASE.

OUR PRESCRIPTION CLOSELY RESEMBLES DEFINING LIOUVILLE THEORY AT FIXED AREA (IN WAKIMOTO REPRESENTATION $i$ IS SIMILAR TO THE AREA OPERATOR).